We propose a theory linking imperfect information to resource misallocation and hence to aggregate productivity and output. In our setup, firms look to a variety of noisy information sources when making input decisions. We devise a novel empirical strategy that uses a combination of firm-level production and stock market data to pin down the information structure in the economy. Even when only capital is chosen under imperfect information, applying this methodology to data from the US, China, and India reveals substantial losses in productivity and output due to the informational friction. Our estimates for these losses range from 7-10% for productivity and 10-14% for output in China and India, and are smaller, though still significant, in the US. Losses are substantially higher when labor decisions are also made under imperfect information. We find that firms turn primarily to internal sources for information; learning from financial markets contributes little, even in the US.

JEL Classifications: O11, O16, O47, E44

Keywords: productivity, misallocation, imperfect information, information in stock prices

*We thank the editor Robert Barro and several anonymous referees, Jaroslav Borovicka, John Haltiwanger, Virgiliu Midrigan, Pete Klenow and Laura Veldkamp for their helpful comments, Andy Atkeson, Yongs Shin, Jennifer La’O, Ben Moll and Bernard Dumas for their insightful discussions of earlier versions, Cynthia Yang for excellent research assistance, and many seminar and conference participants. Joel David gratefully acknowledges financial support from the Center for Applied Financial Economics at USC. joeldavi@usc.edu, hopen@econ.ucla.edu, vvenkate@gmail.com.
I. INTRODUCTION

In a frictionless environment, the optimal allocation of factor inputs across productive units requires the equalization of marginal products. Deviations from this outcome represent a misallocation of resources and translate into sub-optimal aggregate outcomes, specifically, depressed levels of productivity and output. A recent body of work empirically documents the presence of substantial misallocation and points out its potentially important role in accounting for large observed cross-country differences in productivity and income per-capita. With some notable exceptions, however, the literature has remained largely silent about the underlying factors driving this misallocation.

In this paper, we propose just such a theory, linking imperfect information to resource misallocation and hence to aggregate productivity and output. Our point of departure is a standard general equilibrium model of firm dynamics along the lines of Hopenhayn (1992). The key modification here is that firms choose inputs under limited information about their idiosyncratic fundamentals, specifically, demand conditions in their own markets. This informational friction leads to a misallocation of factors across firms in an ex-post sense, reducing aggregate productivity and output.\(^1\) The size of this effect depends on the residual uncertainty at the time of the input choice, which, in turn, is a function of the volatility of the fundamental shocks and the quality of information at the firm level. Our analytical framework enables a sharp characterization of these relationships and yields simple closed-form expressions linking informational parameters at the micro-level to aggregate outcomes.

The second piece of our theoretical framework focuses on the firm’s learning problem. Our flexible information structure assumes that firms learn from a variety of sources. In addition to those within the firm (which we will refer to as ‘private information’), firms also observe movements in their own stock prices, which aggregate the information of financial market participants about the firm’s future prospects. We capture this aggregation using an explicit model of financial market trading in the noisy rational expectations paradigm of Grossman and Stiglitz (1980).\(^2\) Informed investors and noise traders trade shares of the firm’s stock, generating imperfectly revealing equilibrium prices.

The presence of informative asset prices serves two purposes in our analysis: first, as we describe next, the informational content of observed market prices is at the core of our empirical approach and allows us to identify the severity of otherwise unobservable informational frictions. Second, we are able to quantitatively evaluate the contribution of financial markets to allocative efficiency through an informational channel.

---

1. ‘Misallocation’ is somewhat of a misnomer in our environment, as firms are acting optimally given the information at hand. We follow the literature and use the term to refer broadly to deviations from marginal product equalization.
2. We rely particularly on recent work by Albagli, Hellwig, and Tsyvinski (2011b) for our specific modeling structure.
i.e., by providing useful information to decision-makers within firms. Our analysis is, to the best of our knowledge, the first to measure and shed light on the aggregate consequences of this channel in a standard macroeconomic framework.

Any attempt at quantifying uncertainty runs into an obvious difficulty - the econometrician seldom observes agents’ information directly. One approach would be to use the observed degree of misallocation, i.e., dispersion in marginal products. This would lead to an accurate measure of uncertainty if misallocation was entirely driven by informational frictions. In reality, however, a host of other factors are likely to contribute to dispersion in marginal products. To capture these factors, we introduce idiosyncratic output and capital ‘distortions’ a la Hsieh and Klenow (2009) and highlight the difficulty of disentangling the severity of informational frictions from these other factors.

One of the contributions of this paper is a novel empirical strategy that is robust to these concerns, i.e., that gives accurate (or conservative) inferences regarding uncertainty even in the presence of other distortions. The main insight behind this strategy is that we can draw sharp inferences about the degree of uncertainty faced by a firm by observing a subset of its information set. Stock market prices allow us to do just that. The first step is to measure the informational content of prices and the extent to which input decisions covary with them. The correlation of stock returns with future fundamentals and investment, respectively, are natural and intuitive candidates. The second and key step is to note that for a given level of noise, the extent to which firm decisions comove with the price signal is a function of the overall quality of their information (from all sources, including those we do not observe). The poorer this quality, the higher is the correlation between investment and stock market returns. Intuitively, the information contained in stock returns plays a bigger role in the firm’s investment decision when the firm is more uncertain. It is worth emphasizing the need to analyze these moments together - the correlation of returns with investment alone does not tell us much about the extent of uncertainty.

For two special cases of our model - when firm-level fundamentals are i.i.d. or follow a random walk - this intuition can be formalized quite sharply. In the absence of other distortions, we prove that the informational parameters of our model are identified by these two correlations (of stock returns and fundamentals and investment, respectively, as well as the volatility of returns in the random walk case). We further exploit the tractability of these cases to demonstrate the validity of our approach in the presence of other distortions. First, we show that our measure of uncertainty is unaffected by factors or distortions that are correlated with...

---

3. To cite a few examples, these could be technological factors (e.g., adjustment costs), financial constraints, taxes or regulations.
4. We measure investment as the net change in the firm’s capital stock.
5. To give a simple example, the correlation between returns and investment can be high either because firms and investors are both perfectly informed, in which case all firm-level variables are functions of a single fundamental shock, or alternatively, because firms are poorly informed and therefore learn much from market prices.
firm-level fundamentals and so have a ‘scaling’ effect on the firm’s capital choice (i.e., dampen or amplify the responsiveness of investment to perceived fundamentals). Next, our approach is conservative in the presence of distortions that are uncorrelated with firm fundamentals yet still influence the firm’s capital choice. In other words, our empirical strategy gives accurate or conservative estimates of the extent of uncertainty even without explicitly accounting for (and quantitatively disciplining) the host of other factors that in all likelihood lead to observed marginal product dispersion.

Our identification strategy is also robust to the presence of correlation between firm and market information. We prove this result to be exact in the special cases: here, our measure of firm uncertainty remains valid for any degree of correlation. In fact, even if the information in stock prices is fully redundant from the perspective of the firm, implying that firms do not learn anything new from them, our strategy uncovers the true extent of uncertainty. This is a reassuring finding because it shows that our estimates of the severity of informational frictions are robust to assumptions about the extent of commonality between firm and market information sets, an object which is difficult to measure given the richness of information flows between firms and market participants (financial statements, announcements, regulatory filings, etc.). As a last robustness exercise, we use these special cases to assess the potential effects of measurement error on our inferences regarding uncertainty.

In our quantitative work, we depart from these polar cases and consider an intermediate level of persistence in fundamentals that is in line with the data. However, we show numerically that the informational parameters are still well identified by the same set of moments and that our estimates of uncertainty display a similar robustness to perturbations. In particular, we demonstrate this robustness by considering the effect of capital adjustment costs, correlation in firm and market signal errors, and measurement error.

We apply our methodology to data on publicly traded firms from 3 countries - the US, China and India. Our results show substantial uncertainty at the micro level, particularly in China and India, leading to significant levels of misallocation. Even in the US, which has the highest degree of learning, our most conservative estimate for the variance of the firm’s forecast error is about 40% of the ex-ante uncertainty.\footnote{Given our AR(1) structure on fundamentals, the ex-ante uncertainty is simply the variance of contemporaneous innovations.} The corresponding estimates for the other two countries range from about 60-80%. These findings reflect the cross-country variation in the correlation of stock returns with fundamentals and investment - in particular, firm investment decisions in the US covary less with market prices relative to their informational content. The opposite is true in China and India, leading us to conclude that firm-level information must be of relatively higher quality in the US than in the latter two countries.

To put our results in context, we compare them to direct measures of misallocation in our sample and
find that informational frictions account for anywhere from 20-50% of observed dispersion in the marginal (revenue) product of capital, a fraction that goes up once we control for firm-fixed effects. The associated implications for aggregate productivity and output are then quite significant. In China and India, TFP losses (relative to the first best) are in the range of 7-10%, while losses in steady state output (again relative to the first best) range from 10% to almost 15%. The corresponding values in the US are noticeably smaller but still significant - 4% for productivity and 5% for output. Importantly, these baseline calculations assume that only investment decisions are made under imperfect information while labor can adjust perfectly to contemporaneous conditions. In this sense, they are conservative estimates of the total impact of informational frictions. Assuming that the friction affects labor inputs to the same degree as capital leads to losses that are substantially higher. For example, in this case, the gap between status quo and first best increases to about 55-80% in TFP and 80-100% in output for China and India. We interpret this as an upper bound on the total effect of the friction, with reality likely falling somewhere in between this and the baseline version.  

Our framework also enables us to quantify the sources of learning, particularly the contribution of financial markets. To the extent that prices reflect information not otherwise available from firms’ internal sources, stock markets provide firms with valuable information and guide real activity. Note that this does not require investors to know more than firms; only that they are privy to different information that may also be relevant for firm decisions. Here, we arrive at a striking conclusion - learning from stock prices is at best only a small part of total learning at the firm level, even in a relatively well-functioning financial market like the US. Thus, the contribution of financial markets to overall allocative efficiency and aggregate performance through this channel is quite limited. We show that this is primarily due to the high levels of noise in market prices, making them relatively poor signals of fundamentals. A counterfactual experiment delivering access to US-quality financial markets (in a purely informational sense) to firms in China and India generates only small improvements in allocative efficiency. In contrast, a significant amount of learning occurs from private sources, i.e., those internal to the firm. Moreover, disparities along this dimension, that is, in the quality of such information, are the primary drivers of cross-country differences in the severity of informational frictions, much more so than access to well-functioning financial markets. This finding, in

7. We provide suggestive evidence that this is the case using employment data for US firms.
8. The informational content of stock returns and their role in guiding real activity is the focus of an active body of work in corporate finance, both theoretical and empirical. We will briefly survey this literature later in this section.
9. This is true even under our baseline assumption that the information in stock prices is conditionally independent of the firm’s own signals, which, in a sense, is the most optimistic case. Allowing for correlation would further reduce the extent of new information in prices.
10. This is related to the concepts of Forecasting Price Efficiency (FPE), namely, to what extent do prices reflect and predict fundamentals, and Revelatory Price Efficiency (RPE), namely, to what extent do prices promote real efficiency by providing new information to firms, as defined in Bond, Edmans, and Goldstein (2012). Our results imply that the poor RPE of stock prices stems from their poor FPE.
11. Of course, this does not consider other channels through which informative prices, and more generally, well-functioning stock markets may improve efficiency. See the discussion in Section IV.D.
spirit, parallels those in Bloom, Eifert, Mahajan, McKenzie, and Roberts (2013), who highlight the role of manager skill and/or better management practices in explaining cross-country differences in performance. Finally, we show that differences in the volatility of firm-level fundamentals also play a meaningful role in explaining cross-country differences in the severity of informational frictions. Firms in China and India are subject to larger shocks to fundamentals than firms in the US, making the inference problem more difficult in those countries even without the differences in signal qualities.

Before concluding, we provide further evidence that our empirical strategy is indeed picking up micro-level uncertainty. This is not a straightforward exercise - as we have emphasized, it is not obvious how to construct direct measures of information. We perform two experiments aimed at analyzing disaggregated patterns in our estimates of uncertainty: first, we construct an independent direct indicator of firm-level uncertainty by sector in the US using data on firms’ own forecasts of future fundamentals. We show that our estimates at the sectoral level are in line with this alternative indicator. Second, we use our methodology to construct a time-series of micro uncertainty in the US over the last 15 years. The series shows cyclical patterns that are consistent with the findings of other studies12 - specifically, uncertainty rises (falls) during recessions (expansions). We view these results as indirect validation of our methodology.

Our paper relates to several existing branches of literature. We bear a direct connection to recent work on the aggregate implications of misallocated resources, for example, Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Guner, Ventura, and Xu (2008), and Bartelsman, Haltiwanger, and Scarpetta (2013). Indeed, we can map our measure of informational frictions directly into the measures of misallocation studied in these papers, i.e., into the dispersion in marginal products and the covariance between firm-level fundamentals (productivity, for example) and activity (i.e., factor use or output). We differ from these papers in our explicit modeling of a specific friction as the source of misallocation, a feature we share with Midrigan and Xu (2013), Moll (2014), Buera, Kaboski, and Shin (2011), and Asker, Collard-Wexler, and De Loecker (2012), who study the role of financial frictions and capital adjustment costs, respectively. We view our analysis as complementary to those investigating the role of adjustment costs; specifically, in predicting a sluggish response of investment to fundamentals, our theory provides a foundation for adjustment costs, one that may help guide future modeling strategies along these lines and takes a step toward disentangling technological frictions from informational ones, factors that standard estimates of adjustment costs would seem to confound.13 As an example, Asker, Collard-Wexler, and De Loecker (2012) assume in their baseline specification that the adjustment cost technology is the same for all countries. Our analysis provides a primitive that would justify considering cross-country differences in these costs. Our focus on the role of imperfect

12. See, for example, Jurado, Ludvigson, and Ng (2013).
13. Fuchs, Green, and Papanikolaou (2013) investigate another type of informational friction as a foundation for adjustment costs, namely, adverse selection in the market for existing capital.
information is related to that of Jovanovic (2013), who studies an overlapping generations model where informational frictions impede the efficient matching of entrepreneurs and workers, and to Bachmann and Elstner (2013), who use a not dissimilar framework and a survey of German firms to study the importance of expectational biases on factor misallocation and welfare.

Our structure of firm learning holds some similarity with Jovanovic (1982) and our linking of financial markets, information transmission, and real outcomes is reminiscent of Greenwood and Jovanovic (1990). The informational role of financial markets has been the subject of much study, dating back at least to Tobin (1982). We discuss a few particularly relevant examples below and refer the reader to Bond, Edmans, and Goldstein (2012) for an excellent survey. One strand of this literature focuses on measuring the informational content of stock prices. Durnev, Morck, Yeung, and Zarowin (2003) show that firm-specific variation in stock returns, i.e., ‘price-nonsynchronousity,’ is useful in forecasting future earnings and Morck, Yeung, and Yu (2000) find that this measure of price informativeness is higher in richer countries. A related body of work closer to our own and recently surveyed by Bond, Edmans, and Goldstein (2012) looks directly at the feedback from stock prices to investment and other decisions. Chen, Goldstein, and Jiang (2007), Luo (2005), and Bakke and Whited (2010) are examples of studies that find evidence of managers learning from markets while making investment decisions. Bai, Philippon, and Savov (2013) combine a simple investment model with a noisy rational expectations framework to assess whether US stock markets have become more informative over time. Our analysis complements these papers by placing information aggregation through financial markets into a standard macroeconomic setting, which allows us to make precise statements about the quantitative importance of this channel for information transmission, real activity, and aggregate outcomes. Our results on the limited role for stock market information bear some resemblance to the well-known results in Morck, Shleifer, Vishny, Shapiro, and Poterba (1990), who find a limited incremental role for stock prices in predicting investment, once fundamentals are controlled for.14 On the one hand, our analysis can be viewed as providing a structural interpretation of their results; but more importantly, it allows us to quantify the implications for resource allocations and aggregate outcomes.

The paper is organized as follows. Section II describes our model of production and financial market activity under imperfect information. Section III spells out our approach to identifying informational frictions using the two analytically tractable special cases, while Section IV details our numerical analysis and presents our quantitative results. Section V provides additional corroboration of our measurement of uncertainty. We summarize our findings and discuss directions for future research in Section VI. Details of derivations and data work are provided in the Appendix.

14. More precisely, they find very small improvements in $R^2$ when adding stock returns to an investment regression that already includes fundamentals.
II. The Model

In this section, we develop our model of production and financial market activity under imperfect information. We turn first to the production side of the economy, where we derive sharp relationships linking the extent of micro level uncertainty to aggregate outcomes. Next, we flesh out the information structure, including a fully specified financial market in which dispersed private information of investors and noise trading interact to generate imperfectly informative price signals. In order to focus on the particular role of uncertainty, we begin with a simple framework in which imperfect information is the only friction; we then introduce other factors or ‘distortions’ that also contribute to misallocation, making it difficult to identify informational frictions. In Section III, we outline a strategy using both production and stock market data which allows us to overcome this difficulty.

II.A. Production

We consider a discrete time, infinite-horizon economy, populated by a representative large family endowed with a fixed quantity of labor supplied inelastically. The aggregate labor endowment is denoted by $N$. The household has preferences over consumption of a final good and rents capital to firms. The household side of the economy is deliberately kept simple as it plays a limited role in our analysis.

Technology. A continuum of firms of fixed measure one, indexed by $i$, produce intermediate goods using capital and labor according to

$$Y_{it} = K_{it}^{\hat{\alpha}_1} N_{it}^{\hat{\alpha}_2}, \quad \hat{\alpha}_1 + \hat{\alpha}_2 \leq 1$$

These intermediate goods are bundled to produce the single final good using a standard CES aggregator

$$Y_t = \left( \int A_{it} Y_{it}^{\theta - 1} \, di \right)^{\frac{\theta}{\theta - 1}}$$

where $\theta \in (0, \infty)$ is the elasticity of subsitution and $A_{it}$ represents an idiosyncratic demand shifter. This is the only source of fundamental uncertainty in the economy (i.e., we abstract from aggregate risk). We assume that $A_{it}$ follows an AR(1) process in logs:

$$a_{it} = (1 - \rho) \bar{a} + \rho a_{it-1} + \mu_{it}, \quad \mu_{it} \sim \mathcal{N}(0, \sigma^2_{\mu})$$

where we use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., $a_{it} = \log A_{it}$. In this specification, $\bar{a}$ represents the unconditional mean of $a_{it}$, $\rho$ the persistence, and $\mu_{it}$ an i.i.d. innovation
with variance $\sigma^2_\mu$.

**Market structure and revenue.** The final good is produced by a competitive firm under perfect information. This yields a standard demand function for intermediate good $i$

$$Y_{it} = \mathcal{P}_{it}^{-\theta} A_{it}^\theta Y_t \quad \Rightarrow \quad \mathcal{P}_{it} = \left( \frac{Y_{it}}{Y_t} \right)^{1/{\theta}} A_{it}$$

where $\mathcal{P}_{it}$ denotes the relative price of good $i$ in terms of the final good, which serves as numeraire. Revenues are

$$\mathcal{P}_{it} Y_{it} = Y_{it}^{1/{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2}$$

where

$$\alpha_j = \left(1 - \frac{1}{\theta}\right) \hat{\alpha}_j, \quad j = 1, 2$$

This framework accommodates two alternative interpretations of the idiosyncratic component $A_{it}$: as a firm-specific shifter of either demand or productive efficiency. Neither the theory nor our empirical strategy requires us to differentiate between the two, so we will simply refer to $A_{it}$ as a firm-specific fundamental.

**Input choices under imperfect information.** The key element of our theory is the effect of imperfect information on the firm’s choice of factor inputs, that is, capital and labor. For now, they are modeled as static and otherwise frictionless decisions, i.e., firms rent capital and/or hire labor period-by-period, but with potentially imperfect knowledge of their fundamental $A_{it}$.\(^{15}\) Clearly, the impact of the informational friction will depend on whether it affects both inputs or just one. Rather than take a particular stand on this important issue regarding the fundamental nature of the production process, we present results for two cases: in case 1, both factors of production are chosen simultaneously under the same (imperfect) information set; in case 2, only capital is chosen under imperfect information whereas labor is freely adjusted after the firm perfectly learns the current state.

**Case 1: Both factors chosen under imperfect information.** In this case, the firm’s profit-maximization problem is given by

$$\max_{K_{it}, N_{it}} Y_{it}^{1/{\theta}} \mathbb{E}_{it} [A_{it}] K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} - R_t K_{it}$$

---

15. We relax this assumption later in this section and introduce additional distortions into the firm’s input choices.
where $E_{it}[A_{it}]$ denotes the firm’s expectation of fundamentals conditional on its information set $I_{it}$, which we make explicit below. Standard optimality and market clearing conditions imply

$$(4) \quad \frac{N_{it}}{K_{it}} = \frac{\alpha_2 R_t}{\alpha_1 W_t} = \frac{N}{K_t}$$

i.e., the capital-labor ratio is constant across firms.

Our empirical analysis uses moments of firm-level investment data and with this in mind, we use the optimality conditions characterized in (4) to rewrite (3) simply as a capital input choice problem:

$$(5) \quad \max_{K_{it}} \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\theta}} E_{it}[A_{it}] K_{it}^{\alpha_1} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) R_t K_{it}$$

where

$$\alpha = \alpha_1 + \alpha_2 = (\hat{\alpha}_1 + \hat{\alpha}_2) \left( \frac{\theta - 1}{\theta} \right)$$

Notice that the firm’s expectation of revenues depend only on the aggregate capital-labor ratio, its expected $A_{it}$, and the chosen level of capital. The curvature parameter $\alpha$ depends both on the returns to scale in production as well as on the elasticity of demand, and will play an important role in our quantitative analysis below. Solving this problem and imposing capital market clearing gives the following expression for the firm’s capital choice (the labor choice exactly parallels that of capital):

$$(6) \quad K_{it} = \frac{\left( E_{it}[A_{it}] \right)^{\frac{1}{1-\alpha_2}}}{\int \left( E_{it}[A_{it}] \right)^{\frac{1}{1-\alpha_2}} dt} K_t$$

**Case 2: Only capital chosen under imperfect information.** The firm’s problem now is

$$\max_{K_{it}} \left( E_{it} \max_{N_{it}} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} \right) - R_t K_{it}$$

Using the optimality condition for $N_{it}$,

$$(7) \quad N_{it} = \left( \frac{\alpha_2}{W_t} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{\alpha_2}}$$

to substitute for $N_{it}$ in the objective, we can write the capital choice problem as:

$$(8) \quad \max_{K_{it}} \left( 1 - \alpha_2 \right) \left( \frac{\alpha_2}{W_t} \right)^{\frac{1}{\alpha_2}} Y_t^{\frac{1}{\theta}} A_{it}^{\frac{1}{\alpha_2}} E_{it}[\hat{A}_{it}] K_{it}^{\hat{\alpha}} - R_t K_{it}$$
where
\[
\tilde{A}_{it} = A_{it}^{\frac{1}{1-\alpha}}, \quad \tilde{\alpha} = \frac{\alpha_1}{1-\alpha_2}
\]

Thus, the firm’s capital choice problem here has the same structure as in case 1 (compare equations (5) and (8)), but with a slightly modified fundamental and overall curvature. This makes the two cases qualitatively very similar, though, as we will see, the quantitative implications will be quite different. We mark with a \( \sim \) the transformed objects that are relevant in case 2, a convention we will carry throughout this section.

To complete our characterization of the firm’s problem and therefore of the production-side equilibrium in the economy, we need to spell out the firm’s information set \( \mathcal{I}_{it} \). We defer this discussion to the following subsection and for now directly make conjectures about firm beliefs, which we will later show to be true. Specifically, we guess (and later verify) that the conditional distribution of the fundamental is log-normal in both cases, i.e.,

\[
\begin{align*}
    a_{it} | \mathcal{I}_{it} & \sim \mathcal{N} \left( \mathbb{E}_{it} [a_{it}], \mathbb{V} \right) \\
    \tilde{a}_{it} | \mathcal{I}_{it} & \sim \mathcal{N} \left( \mathbb{E}_{it} [\tilde{a}_{it}], \tilde{\mathbb{V}} \right)
\end{align*}
\]

where \( \mathbb{E}_{it} [a_{it}] \) and \( \mathbb{V} \) denote the posterior mean and variance in case 1, respectively, and similarly \( \mathbb{E}_{it} [\tilde{a}_{it}] \) and \( \tilde{\mathbb{V}} \) in case 2. Equivalently, the forecast error of firm \( i \) at time \( t \), \( a_{it} - \mathbb{E}_{it} [a_{it}] \), is a mean-zero variable with variance \( \mathbb{V} \) (or \( \tilde{\mathbb{V}} \) in case 2). Focusing on case 1 for a moment, and invoking the usual convention that a law of large numbers applies to the cross-sectional distribution, this implies that the distribution of the forecast error in the economy is also normal, centered around 0 with variance \( \mathbb{V} \). We can then show that \( \mathbb{V} \) maps directly into the dispersion of the marginal revenue product of capital (in logs),

\[
\sigma^2_{mrpk} = \mathbb{V}
\]

Similarly, it has a negative effect on the covariance between fundamentals and firm activity as examined, for example, in Bartelsman, Haltiwanger, and Scarpetta (2013) and Olley and Pakes (1996), i.e., the covariance between \( a_{it} \) and \( k_{it} \) satisfies \( \sigma_{ak} = \frac{\sigma^2 - \mathbb{V}}{1-\alpha} \). An analogous correspondence holds for case 2.

Thus, \( \mathbb{V} \) indexes the severity of informational frictions in the economy. When investment decisions are distorted only by uncertainty, \( \mathbb{V} \) is a sufficient statistic for misallocation and the associated productivity/output losses. When we introduce other distortions below, \( \mathbb{V} \) will continue to summarize the effects of uncertainty, though \( \sigma^2_{mrpk} \) and \( \sigma_{ak} \) will also reflect the effects of these other factors.
Aggregation. We now turn to the aggregate economy, and in particular, output and measures of total factor productivity (TFP). Given our focus on misallocation, we abstract from aggregate risk and restrict our attention to a stationary equilibrium, in which all aggregate variables remain constant through time. From here on, we will also assume constant returns to scale in production, i.e., \( \hat{\alpha}_1 + \hat{\alpha}_2 = 1 \). Though not essential to our results, this greatly simplifies the expressions.\(^{16}\) Relegating the lengthy but straightforward derivations to Appendix I.A and I.B, we are able to derive a simple representation for aggregate output:

\[
\log Y \equiv y = a + \hat{\alpha}_1 k + \hat{\alpha}_2 n
\]

Aggregate TFP, denoted \( a \), is endogenous and is given by

\[
\text{Case 1:} \quad a = a^* - \frac{1}{2} \theta \hat{V} \\
\text{Case 2:} \quad a = a^* - \frac{1}{2} (\theta \hat{\alpha}_1 + \hat{\alpha}_2) \hat{\alpha}_1 \hat{V}
\]

where

\[
a^* = \frac{\theta}{\theta - 1} \pi + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma_a^2}{1 - \alpha}
\]

is aggregate TFP under full information, which is identical in the two cases.

These expressions are at the heart of our mechanism and reveal a sharp connection between the micro-level uncertainty summarized by \( V \) (or \( \hat{V} \)) and aggregate TFP: in both cases, aggregate productivity monotonically decreases in uncertainty, with the magnitude of the effect depending on the elasticity of substitution (i.e., the degree of curvature) \( \theta \). The higher is \( \theta \), that is, the closer we are to perfect competition, the more severe are the losses from misallocated resources. The intuition is easy to see - when goods are highly substitutable, misallocation is particularly costly. In case 1, only \( \theta \) plays a role. In case 2, the relative shares of capital and labor in the production function also matter. Intuitively, the greater is labor’s share \( \hat{\alpha}_2 \) (and so the lower capital’s share \( \hat{\alpha}_1 \)), the greater the ability of firms to mitigate the effects of imperfect capital choice by adjusting labor. To take the extreme case, as \( \hat{\alpha}_2 \) approaches one and so \( \hat{\alpha}_1 \) zero, the multiplier on \( \hat{V} \) approaches zero, that is, the informational friction has no effect on aggregate TFP. This flexibility is absent in case 1, in which both inputs are subject to the same friction. Notice that the two cases are equivalent at the opposite extreme, i.e. when \( \hat{\alpha}_1 = 1 \) and \( \hat{\alpha}_2 = 0 \). It is easy to see that for a fixed set of parameters, the coefficient on uncertainty in case 2 is smaller than that in case 1.

Holding the aggregate factor stocks fixed, the effect of informational frictions on aggregate productivity

\(\text{\textsuperscript{16}}\) It is straightforward to relax this assumption and work in the more general case; we do so in our derivations in the Appendix.
a is also the effect on aggregate output \( y \). However, the misallocation induced by informational frictions also reduces incentives for capital accumulation and so the steady state stock of capital decreases with uncertainty. Incorporating this additional effect, we obtain:

\[
\begin{align*}
\frac{dy}{dV} &= \frac{da}{dV} \left( \frac{1}{1 - \bar{\alpha}_1} \right) \\
\frac{dy}{d\tilde{V}} &= \frac{da}{d\tilde{V}} \left( \frac{1}{1 - \bar{\alpha}_1} \right)
\end{align*}
\]  

\text{II.B. Information}

In the previous subsection, we derived a sufficient statistic \( V \) for the impact of informational frictions on resource misallocation and the resulting consequences for aggregate outcomes. We now describe the information structure, that is, the elements of the firm’s information set \( \mathcal{I}_{it} \), and characterize \( V \) in terms of primitives - specifically, the variances of fundamentals and signal errors.

The firm’s information set \( \mathcal{I}_{it} \) has three elements. The first is the entire history of its fundamental shock realizations, i.e., \( \{a_{it-s}\}_{s=1}^{\infty} \) - in the context of our model, \textit{ex-post} revenues reveal the fundamental perfectly.\textsuperscript{17} Second, the firm also observes a noisy private signal of its contemporaneous fundamental

\[ s_{it} = a_{it} + e_{it}, \quad e_{it} \sim \mathcal{N}(0, \sigma^2_e) \]

where \( e_{it} \) is an i.i.d. mean-zero and normally distributed noise term.

The third and last element of the firm’s information set is the price of its own stock, \( P_{it} \). The final piece of our theory then is to outline how the stock price is determined and to characterize its informational content. We will show that, under our assumptions about the structure of financial markets, the stock price is informationally equivalent to a signal of the form

\[ \tilde{s}_{it} = a_{it} + \sigma_v z_{it} \]

where \( z_{it} \sim \mathcal{N}(0, \sigma^2_z) \) and is independent of \( a_{it} \) and \( e_{it} \), and \( \sigma_v \) is a scalar. In other words, from an informational standpoint, the stock price \( P_{it} \) is a conditionally independent, normally distributed signal of firm fundamentals of precision \( \frac{1}{\sigma^2_v \sigma^2_e} \). Formally, the firm’s information set can be summarized by \( \mathcal{I}_{it} = (a_{it-1}, s_{it}, \tilde{s}_{it}) \). It is easily verified that the conditional and cross-sectional distributions are log-normal under this information set, exactly as conjectured. Direct application of Bayes’ rule implies the following formula

\textsuperscript{17. In our numerical analysis, we interpret a period as relatively long (3 years), making this a very natural assumption.}
for the conditional expectation of the fundamental $a_{it}$,

$$E_{it}[a_{it}] = \frac{V}{\sigma^2_\mu} [(1 - \rho) \bar{a} + \rho a_{it-1}] + \frac{V}{\sigma^2_e} s_{it} + \frac{V}{\sigma^2_v \sigma^2_z} \tilde{s}_{it}$$

where $V$ is the posterior variance given by:

$$V = \left( \frac{1}{\sigma^2_\mu} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_v \sigma^2_z} \right)^{-1}$$

Thus, $V$, our sufficient statistic, is increasing in the noisiness of the two signals, private and market ($\sigma^2_e$ and $\sigma^2_v \sigma^2_z$, respectively). In the absence of any learning, $V = \sigma^2_\mu$, that is, all fundamental uncertainty remains unresolved at the time of the firm’s input choice. At the other extreme, under perfect information, $V = 0$.

The following subsection presents a noisy rational expectations model of stock market trading in the spirit of Grossman and Stiglitz (1980). It maps $\sigma^2_v$ and $\sigma^2_z$ to the quality of investor information and the extent of noise trading, respectively, and generates precisely the signal in (15). It also contains an explicit characterization of the price function. These details will prove useful for the numerical analysis in Section IV, but are not essential to follow the identification arguments in Section III, so an impatient reader can skip ahead to Section II.C.

**The stock market.** Our specific model structure in this subsection draws heavily from recent work by Albagli, Hellwig, and Tsyvinski (2011a) and Albagli, Hellwig, and Tsyvinski (2011b). For each firm $i$, there is a unit measure of outstanding stock or equity, representing a claim on the firm’s profits. These claims are traded by two groups of agents - imperfectly informed investors and noise traders.

There is a unit measure of risk-neutral investors for each stock. Every period, each investor decides whether or not to purchase up to a single unit of firm $i$’s stock at the current market price $P_{it}$. This assumption is standard in the literature; without it, risk neutral investors would take unbounded positions in the stock. The market is also populated by noise traders who purchase a random quantity $\Phi (z_{it})$ of stock $i$ each period, where $z_{it} \sim N(0, \sigma^2_z)$ is i.i.d. and $\Phi$ denotes the standard normal CDF. This convenient transformation ensures that the total demand of these traders is positive and less than one, the total supply.

Like firms, investors also observe the entire history of fundamental realizations, and in particular, know $a_{it-1}$ at time $t$. They also see the current stock price $P_{it}$, or equivalently, place limit orders conditional

---

18. There are several possible interpretations of the precise nature of these agents. One is that they are intermediaries investing on behalf of the representative family. Some of them are rational, optimizing investors, while others trade randomly. Because the household cannot distinguish between the two, they co-exist. An alternative is that they are members of the representative large family, again, with some members trading rationally and some randomly. The exact interpretation of these agents in the model is not crucial for our purposes - only that some trade rationally given their information and others randomly, with the net result that prices only imperfectly reflect firm fundamentals.
on $P_{it}$. Finally, each investor $j$ is endowed with an independent noisy private signal about the firm’s contemporaneous fundamental $a_{it}$:

$$s_{ijt} = a_{it} + v_{ijt}, \quad v_{ijt} \sim \mathcal{N}(0, \sigma_v^2)$$

Our baseline assumption is that the random variables $v_{ijt}$ and $z_{it}$ are independent of the fundamental $a_{it}$ and the noise in the firm’s private signal.\(^{19}\) Aggregating demand of both investors and noise traders, the market clearing condition is

$$\int d (a_{it-1}, s_{ijt}, P_{it}) dF (s_{ijt}|a_{it}) + \Phi (z_{it}) = 1$$

where $d (a_{it-1}, s_{ijt}, P_{it}) \in [0, 1]$ is the demand of investor $j$ and $F$ is the conditional distribution of investors’ private signals.

The expected payoff to investor $j$ from purchasing the stock is given by

$$E_{ijt}[\Pi_{it}] = \int \left[ \pi (a_{it-1}, a_{it}, P_{it}) + \beta \overline{P} (a_{it}) \right] dH (a_{it}|a_{it-1}, s_{ijt}, P_{it})$$

The term $\pi (a_{it-1}, a_{it}, P_{it})$ denotes the expected current profit of the firm as a function of history, the current realization of the fundamental $a_{it}$ and the current stock price $P_{it}$.\(^{20}\) The expected current profit is a function of the current price $P_{it}$ because it enters the firm’s information set and through that, influences firm decisions.\(^{21}\) The distribution $H (a_{it}|a_{it-1}, s_{ijt}, P_{it})$ is investor $j$’s posterior over $a_{it}$ and $\overline{P} (a_{it})$ is the expected price in period $t + 1$, conditional on the current fundamental $a_{it}$. Formally,

$$\overline{P} (a_{it}) = \int \mathbb{P} (a_{it}, a_{it+1}, z_{it+1}) dG (a_{it+1}, z_{it+1}|a_{it})$$

is obtained by integrating the price function $\mathbb{P} (\cdot)$ over $(a_{it+1}, z_{it+1})$ (using the conditional distribution $G$).

Clearly, optimality implies:

$$d (a_{it-1}, s_{ijt}, P_{it}) = \begin{cases} 1 & \text{if } E_{ijt}[\Pi_{it}] > P_{it} \\ \in [0, 1] & \text{if } E_{ijt}[\Pi_{it}] = P_{it} \\ 0 & \text{if } E_{ijt}[\Pi_{it}] < P_{it} \end{cases}$$

\(^{19}\) This implies that the noise in the stock price is orthogonal to the firm’s information and in this sense, maximizes the potential for learning from prices. We relax this assumption later in our robustness section.

\(^{20}\) Given the assumption of an AR(1) structure for fundamentals and an i.i.d. process for $z_{it}$, the most recent fundamental realization, $a_{it-1}$, is a sufficient statistic for historical information.

\(^{21}\) Appendix I.C explicitly characterizes $\pi (\cdot)$ in terms of the firm’s problem studied in the previous subsection.
that is, an investor purchases the maximum quantity allowed (1 share) when the expected payoff (conditional on her information) strictly exceeds the price, does not purchase any shares when the expected payoff is strictly less than the price, and is indifferent when the two are equal.

A rational expectations equilibrium is then a set of functions for prices $P(\cdot)$, expected profits $\pi(\cdot)$, investor decision rules $d(\cdot)$, and firm capital choice $k(\cdot)$, such that, for any history of shocks, (i) $\pi(\cdot)$ is consistent with firm optimization and the price function $P(\cdot)$; (ii) $d(\cdot)$ is consistent with investor optimality as in (17) above; (iv) $k(\cdot)$ is optimal given the firm’s information set; and (v) markets for capital and each firm’s stock clear.

We conjecture that equilibrium outcomes have the following 2 properties: (a) trading decisions of investors are characterized by a threshold rule, i.e., there is a signal $\hat{s}_{it}$ such that only investors observing signals higher than $\hat{s}_{it}$ choose to buy, and (b) the market price $P_{it}$ is an invertible function of $\hat{s}_{it}$.\footnote{Given that we have unbounded shocks, there are always histories where this conjecture does not hold. However, these realizations are extremely unlikely - in fact, they do not show up at all in any our simulated sample paths. In this sense, we verify this conjecture in our numerical results.}

Aggregating the demand decisions of all investors, market clearing then implies

$$1 - \Phi \left( \frac{\hat{s}_{it} - a_{it}}{\sigma_v} \right) + \Phi (z_{it}) = 1$$

which leads to a simple characterization of the threshold signal

$$\hat{s}_{it} = a_{it} + \sigma_v z_{it}$$

This defines a monotonic relationship between $P_{it}$ and $\hat{s}_{it}$, implying that observing the stock price is informationally equivalent to observing $\hat{s}_{it}$, exactly as in (15). The precision of this signal, $\frac{1}{\sigma^2_v \sigma^2_z}$, is decreasing in both the variance of the noise in investors’ private signals and the size of the noise trader shock. This simple expression for price informativeness is the key payoff of the structure we have imposed on stock market trading. It allows a complete characterization of the firm’s information set and hence the posterior variance $\mathbb{V}$, even without an explicit solution for the price function.

Finally, note that the marginal investor, i.e., the investor whose signal $s_{ijt} = \hat{s}_{it}$, must be exactly indifferent between buying and not buying. It follows then that the price $P_{it}$ must equal her expected payoff from holding the stock:

$$P_{it} = \int [\pi (a_{it-1}, a_{it}, P_{it}) + \beta \hat{P} (a_{it})] \ dH (a_{it} | a_{it-1}, \hat{s}_{it}, P_{it})$$

$$= \int [\pi (a_{it-1}, a_{it}, \hat{s}_{it}) + \beta \hat{P} (a_{it})] \ dH (a_{it} | a_{it-1}, \hat{s}_{it}, \hat{s}_{it})$$
where, with a slight abuse of notation, we replace $P_{it}$ with its informational equivalent $\hat{S}_{it}$. In the Appendix, we rewrite this equation in recursive form and outline the computational procedure we use in our numerical analysis.

**II.C. Other Distortions**

Thus far, we have assumed that the firm’s capital choice is a static and otherwise undistorted decision, which depends only on the firm’s expectations of fundamentals. In logs, we have:

\begin{equation}
    k_{it} = \frac{E_{it} [a_{it}]}{1 - \alpha} + \text{Constant}
\end{equation}

Under this assumption, imperfect information is the only source of misallocation (see, for example, equation (9)). In reality, however, investment decisions may be affected by a number of other factors. These may originate, for example, from technological limitations (e.g., adjustment costs), contracting frictions (e.g., financial constraints), or government policies (e.g., taxes). All of these can lead to dispersion in marginal products.

To capture these factors, we introduce ‘distortions’ drawn from a flexible, albeit stylized, class, into the firm’s capital choice problem. The goal is to incorporate some essential features of these other factors without sacrificing analytical tractability. Specifically, we follow Hsieh and Klenow (2009) and introduce idiosyncratic output and capital distortions into the firm’s objective, which becomes:

\begin{equation}
    (1 - \tau_{Y, it}) A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - WN_{it} - (1 + \tau_{K, it}) RK_{it}
\end{equation}

The three random variables $A_{it}$, $(1 - \tau_{Y, it})$ and $(1 + \tau_{K, it})$ are assumed to be jointly log-normal with covariance matrix $\Sigma$. Importantly, we impose no restrictions on $\Sigma$ - distortions are allowed to covary with fundamentals in an arbitrary way. In Appendix I.D, we show that the firm’s optimality condition can be written (in logs) as

\begin{equation}
    k_{it} = \frac{E_{it} [a_{it} + \tau_{it}]}{1 - \alpha} + \text{Constant}
\end{equation}

where $\tau_{it}$ reflects the combined effects of $\tau_{Y, it}$ and $\tau_{K, it}$ on the firm’s capital choice. This composite distortion $\tau_{it}$ is also jointly normally distributed with $a_{it}$, with a covariance matrix that can be derived from $\Sigma$. Then,

---

23. We use the term ‘distortion’ following Hsieh and Klenow (2009), though we do not take a stand on the true nature of these factors.
without loss of generality, $\tau_{it}$ has the following representation:

$$\tau_{it} = \gamma a_{it} + \varepsilon_{it}, \quad \varepsilon_{it} \sim N\left(0, \sigma^2_{\varepsilon}\right)$$

where $\varepsilon_{it}$ is uncorrelated with $a_{it}$. The parameter $\gamma$, which depends on the elements of the covariance matrix of $\tau_{it}$, indexes the extent to which $\tau_{it}$ is correlated with firm-level fundamentals. For example, if $\gamma < 0$, the distortion discourages (encourages) investment by firms with stronger (weaker) fundamentals. The second term, $\varepsilon_{it}$, captures distortions that are orthogonal to fundamentals. The two parameters $\gamma$ and $\sigma^2_{\varepsilon}$ together pin down the amplitude of these distortions and their correlation with fundamentals.

Finally, it remains to specify the extent to which these distortions are within the firm’s information set $I_{it}$. We assume that firms perfectly observe the uncorrelated component $\varepsilon_{it}$ at the time of making their time $t$ capital choice. They are also aware of the structure of distortions, i.e., they know the parameters $\gamma$ and $\sigma^2_{\varepsilon}$. Under this information structure, equation (19) becomes

$$k_{it} = \frac{(1 + \gamma) E_{it}[a_{it}] + \varepsilon_{it}}{1 - \alpha} + \text{Constant}$$

It is straightforward to derive

$$\sigma^2_{mrpk} = \mathbb{V} + \gamma^2 \left(\sigma^2_a - \mathbb{V}\right) + \sigma^2_{\varepsilon}$$

that is, observed dispersion in marginal products now reflects the effects of both uncertainty (as in equation (9)) and other factors, whether correlated or uncorrelated with fundamentals ($\sigma^2_{mrpk}$ is increasing in $\gamma^2$ and $\sigma^2_{\varepsilon}$). Similarly, the covariance between fundamentals and investment is now

$$\sigma_{ak} = \left(\frac{1 + \gamma}{1 - \alpha}\right) \left(\sigma^2_{\mu} - \mathbb{V}\right)$$

Thus, the potential presence of other distortions confounds the otherwise tight connections between both investment and uncertainty (compare equations (18) and (19)) and marginal product dispersion and uncertainty (compare equations (9) and (20)). This highlights the identification challenge we face: disentangling informational frictions from other factors requires explicit assumptions about the (size and covariance of the) latter. In the next section, we outline an empirical approach that allows us to overcome this challenge and infer the extent of uncertainty without having to take a stand on the properties of the distortions reflected in $\tau_{it}$.  

18
III. IDENTIFYING INFORMATIONAL FRICCTIONS

An obvious hurdle in measuring the extent of uncertainty is that we do not observe agents’ signals, making it difficult to discipline the informational parameters directly. One approach would be to use the model’s implications for observable moments in production-side variables. However, as illustrated in the previous section, those moments confound distortions other than imperfect information with the effects of uncertainty. Thus, a strategy which targets these measures directly (i.e., chooses $V$ to match these moments without adjusting for $\tau_{it}$) would lead to a biased estimate of the severity of informational frictions. For example, inferring $V$ from $\sigma^2_{mrpk}$ overstates the extent of uncertainty, while using $\sigma_{ak}$ can lead to an upward or downward bias, depending on the sign of $\gamma$. Similar concerns also apply to inferences made directly from the cross-sectional variance of investment or revenue - these moments also reflect both uncertainty and other factors, and so using them without taking a stand on the nature of these other distortions can be problematic. Quantifying the role - whether individually or jointly - of all of these factors in determining allocative efficiency is certainly the overall objective of the growing literature on misallocation, but one that is well beyond the scope of this paper. Our more limited goal here is to isolate and quantify the degree of firm-level uncertainty and its particular role in generating misallocation.

To this end, we develop an empirical strategy that allows us to draw robust inferences about uncertainty, even without explicitly accounting for these other factors. Our approach combines moments from firm-level production and stock market data to pin down the informational parameters of our model. Specifically, we use three readily observable moments - the correlations of stock returns with both fundamentals and investment, and the volatility of stock returns. Two key insights underlie this strategy and the choice of moments: first, correlations are invariant to scaling effects, which makes our strategy robust to the presence of other factors that dampen (or exacerbate) the responsiveness of investment to fundamentals; second, the structure of the model allows us to say a lot about the extent of uncertainty by looking at the comovement of investment with any element of the firm’s information set (in our case, this element is stock returns).

In this section, we develop the intuition for that strategy by analyzing two special cases of our model - when firm level shocks are i.i.d. and when they follow a random walk. We exploit the tractability of these two cases to analytically demonstrate the key insights of our approach and choice of moments. In particular, we derive intuitive expressions linking these moments to the informational parameters (as well as

24. Note that we can use the structure of the model to directly back out the fundamental $a_{it}$ from data on value-added and capital as $v_{at} - \alpha k_{it}$.

25. The Gaussian structure of uncertainty plays an important role in allowing us to summarize uncertainty with a single moment and derive a sharp mapping between that object and observable second moments. In a more general stochastic environment, we conjecture that these moments will be informative, though we may need additional ones to fully uncover the extent of uncertainty.
other distortions). We are then able to prove that in these two cases, our strategy (i) correctly identifies the true extent of uncertainty in the absence of other distortions; (ii) correctly identifies the true extent of uncertainty in the presence of correlated distortions; and (iii) provides a lower bound on the true extent of uncertainty in the presence of uncorrelated distortions.

We proceed in two steps. We first derive the identification equations in the absence of other distortions. This will make the underlying intuition very transparent. We then show that the strategy remains valid even in the presence of other distortions. We also highlight some merits of our approach relative to reduced-form, regression-based strategies used extensively in existing work on the relationship between stock markets and investment. Finally, in two additional robustness exercises, we show that our approach retains its validity under a general (and unknown) correlation structure between firm and market information, and explore the potential effects of classical measurement on our estimates.

### III.A. Identification

We begin with the case where shocks to fundamentals are i.i.d., i.e., $\rho = 0$ in equation (1). We then show that our results also hold when fundamentals follow a random walk.

**Without other distortions.** First, assume that there are no other distortions, i.e. $\tau_{it} = 0$. A log-linear approximation of the stock price (around the deterministic case) leads to:

\[ p_{it} \equiv \log P_{it} \approx \kappa (\mu_{it} + \sigma_v z_{it}) + \text{Constant} \]

where $\kappa$ is a constant that depends on the primitive parameters.

Similarly, capital is a log-linear function of the firm’s expectation of the current innovation

\[ k_{it} = \mathbb{E}_{it} [\mu_{it}] + \text{Constant} \]

which is a precision-weighted average of its private signal and the information in prices:

\[ \mathbb{E}_{it} [\mu_{it}] = \phi_1 (\mu_{it} + e_{it}) + \phi_2 (\mu_{it} + \sigma_v z_{it}) \]

26. When we return to our general model in the following section, we will verify numerically that the intuition for the special cases analyzed here extends to the general model used there.

27. See Appendix 1.E for derivations. Intuitively, the price is determined by market expectations of the fundamental.
where $\phi_1 = \frac{V}{\sigma_e^2}$, $\phi_2 = \frac{V}{\sigma_e^2\sigma_z^2}$.

From here, we derive the following expressions for the two correlations of interest, that between returns and changes in fundamentals, denoted $\rho_{pa}$, and between returns and investment, denoted $\rho_{pk}$:

$$\rho_{pa} \equiv \text{Corr} (p_{it} - p_{it-1}, a_{it} - a_{it-1}) = \frac{1}{\sqrt{1 + \frac{\sigma_e^2\sigma_z^2}{\sigma_\mu^2}}}$$

(24)

$$\rho_{pk} \equiv \text{Corr} (p_{it} - p_{it-1}, k_{it} - k_{it-1}) = \frac{1}{\sqrt{\left(1 + \frac{\sigma_e^2\sigma_z^2}{\sigma_\mu^2}\right)\left(1 - \frac{V}{\sigma_\mu^2}\right)}}$$

(25)

Equation (24) shows that the higher is $\frac{\sigma_e^2\sigma_z^2}{\sigma_\mu^2}$, the noise-to-signal ratio in prices, the lower is the correlation of returns with fundamentals. Equation (25) then implies that for a given level of noise in prices, $\rho_{pk}$ is increasing in the firm’s posterior variance $\frac{V}{\sigma_z^2}$ - investment choices covary more strongly with the signal when firms are more uncertain.

As noted above, the fundamental can be directly recovered from data on value added and capital as $a_{it} = v a_{it} - \alpha k_{it}$ where $v a_{it}$ denotes log of value added. This allows us to directly estimate $\sigma_\mu^2$, which combined with the estimate for $\frac{V}{\sigma_z^2}$ from above yields $V$, our sufficient statistic for misallocation. Finally, the definition of $V$ equation (16) allows us to recover $\sigma_z^2$, the noise in the firm’s private signal ($\sigma_e^2\sigma_z^2$ can be obtained from the expression for $\rho_{pa}$ above).

Notice that a high $\rho_{pk}$ is not by itself indicative of the degree of uncertainty. Firm choices can be highly correlated with returns either because they both track fundamentals very closely or because firms are uncertain.$^{29}$ Observing $\rho_{pa}$ allows us to isolate the effect of the latter. To see this more clearly, substitute for $\rho_{pa}$ in (25) to derive

$$\frac{\rho_{pa}}{\rho_{pk}} = \sqrt{1 - \frac{V}{\sigma_\mu^2}} \quad \Rightarrow \quad \frac{V}{\sigma_\mu^2} = 1 - \left(\frac{\rho_{pa}}{\rho_{pk}}\right)^2$$

(26)

Thus, with i.i.d. fundamentals and no other distortions, equation (26) yields a simple way to measure uncertainty using the relative correlation, $\frac{\rho_{pa}}{\rho_{pk}}$.\textsuperscript{30} Under full information, this ratio takes a value of 1. This

---

\textsuperscript{28} From here on, in a slight abuse of notation, we use $V$ to denote the uncertainty in both case 1 and case 2, where it should be understood that $V$ in case 2 corresponds to $\bar{V}$ in the theory. We similarly use $a_{it}$ to denote the fundamental in both cases and $\alpha$ the relevant curvature parameter.

\textsuperscript{29} For example, suppose we make prices more informative, i.e., decrease $\sigma_e^2\sigma_z^2$. Then $\rho_{pk}$ rises even though uncertainty decreases.

\textsuperscript{30} Though we focus on $V$ here, we show in Appendix I.E that all the informational parameters can be separately identified.
sharp link between the relative correlation and the severity of informational frictions guides our empirical
approach in our quantitative analysis below. The more general version of the model precludes an analytical
mapping between these correlations and $V$, but numerical simulations reveal a very similar positive
relationship between the relative correlation and $V$.

**With other distortions.** Next, we examine the validity of the identification strategy outlined above to
the presence of other distortions, i.e., when $\tau_{it} \neq 0$. First, note that in a neighborhood of the undistorted
deterministic optimum, distortions have only second order effects on profits and therefore on stock prices.
As a result, equation (22) is still a good approximation. Consider the case with only correlated distortions,
i.e., $\gamma \neq 0, \sigma_\varepsilon^2 = 0$. If $\gamma < 0 (\gamma > 0)$, the distortion scales down (up) the response of investment to beliefs by
a factor $1 + \gamma$. Since correlations are invariant to such scaling effects, this factor cancels from the expressions
in (24) and (25) and therefore, equation (26) is unaffected - that is, the ratio of the correlations still uncovers
the true $V$. In other words, equation (26) allows us to measure $V$ even without knowing $\gamma$.

Next, consider the case where distortions are uncorrelated with fundamentals, i.e., $\gamma = 0, \sigma_\varepsilon^2 \neq 0$. The
only moment that is affected is $\rho_{pk}$, which is given by

$$
\rho_{pk} = \frac{1}{\sqrt{(1 + \frac{\sigma^2_s}{\sigma^2_\mu})(1 - \frac{V}{\sigma^2_\mu})}} = \frac{\rho_{pa}}{\sqrt{1 - \frac{V}{\sigma^2_\mu}}}
$$

It is straightforward to show that using (26) now underestimates $V$:

$$
1 - \left( \frac{\rho_{pa}}{\rho_{pk}} \right)^2 = \frac{V}{\sigma^2_\mu} - \frac{\sigma^2_\varepsilon}{\sigma^2_\mu} < \frac{V}{\sigma^2_\mu}
$$

In sum, our identification strategy leads us to the correct measure of uncertainty when distortions are
correlated with fundamentals, even without explicitly accounting for those distortions. With uncorrelated
distortions, on the other hand, it leads to a lower bound, i.e., a conservative estimate of the extent of
uncertainty. Note that in both instances (correlated and uncorrelated), if the true $V$ were 0 (i.e., firms
were perfectly informed), using (26) would not lead us to a positive estimate. In other words, we would not
find evidence of uncertainty in a full information economy, even one with dispersion in marginal products.

**Relationship to investment-Q regressions.** We can also use this special case to compare our strategy to
a regression-based approach widely used in the empirical corporate finance literature to analyze the feedback

using the two correlations and the volatility of returns. The latter (in combination with $\rho_{pa}$) allows us to disentangle $\sigma_\varepsilon^2$ and
$\sigma_\mu^2$.

31. However, the effect of uncertainty on aggregate outcomes does depend on $\gamma$.
32. This result holds even if distortions are correlated with errors in the firm’s signal.
between stock prices and investment decisions. For example, Morck, Shleifer, Vishny, Shapiro, and Poterba (1990) regress investment growth on stock returns, sales growth and other controls.\(^{33}\) We begin by noting that investment can be expressed as a log-linear function of fundamentals, signal errors and prices:

\[
\Delta k_{it} = \lambda_1 (\Delta \mu_{it} + \Delta e_{it}) + \lambda_2 \Delta p_{it}
\]

where \(\Delta x_{it} \equiv x_{it} - x_{it-1}\). Our structural model enables us to interpret the coefficients from this reduced-form regression in terms of the informational parameters. For example, consider the coefficient on stock returns, \(\lambda_2\):

\[
\lambda_2 = \frac{1 + \gamma}{(1 - \beta)} \psi \left( \frac{\psi}{\sigma_v^2 \sigma_z^2} \right)
\]

where \(\psi\) is a constant that only depends on informational parameters and is characterized in the Appendix. Ignoring for a moment the \(\gamma\) term, equation (28) reveals the same intuition as in (25): investment can covary strongly with returns (\(\lambda_2\) is high) either because firms are very uncertain, i.e., \(\psi\) is large, and so their beliefs respond strongly to even a noisy signal, or because markets are highly informative, i.e., \(\sigma_v^2 \sigma_z^2\) is low.

The presence of \(\gamma\) distorts the coefficient \(\lambda_2\), making it hard to disentangle the information-related effects from those of other factors. Recall that our approach identifies the true extent of uncertainty even when \(\gamma\) is non-zero. There is another potential concern with the regression implied by (27). The coefficient estimates are consistent only if the errors \(\Delta e_{it}\) are orthogonal to the regressors \(\Delta \mu_{it}\) and \(\Delta p_{it}\). If the noise terms in the signals of firms and investors are correlated, this exogeneity assumption is violated, leading to inconsistent estimates. We will show later in this section that our approach is robust to this concern as well.

**Permanent shocks.** A second special case that lends itself to analytical characterization is that of permanent shocks, i.e., \(\rho = 1\). The main insights from the i.i.d. case extend to this case as well.\(^{34}\) As in the i.i.d. case, we start by deriving the key relationships under the assumption of \(\tau_{it} = 0\) and then examine robustness to the presence of other distortions. In Appendix I.E, we derive the following relationships, which

---

33. Chen, Goldstein, and Jiang (2007) use \(Q\) and cash flow growth as their independent variables.
34. With permanent shocks and no exit, there is no stationary distribution. Since our goal here is primarily to provide intuition for our empirical strategy, we ignore this complication and interpret this as a limiting case.
map the three moments - $\rho_{pk}$, $\rho_{pa}$ and $\sigma^2_p$ - into the informational parameters:

\[
\begin{align*}
\frac{\mathbb{V}}{\sigma^2_\mu} &= \frac{\rho_{pk} - \rho_{pa}}{\eta} \\
\frac{\sigma^2_c}{\sigma^2_\mu} &= \frac{(1 - \eta^2)}{2\rho^2_{pa}} + \frac{\eta}{\rho_{pa} - 1} \\
\frac{\sigma^2_z + 1}{\sigma^2_\mu + \sigma^2_z} &= \frac{1}{\eta}
\end{align*}
\]

where $\eta = \left(\frac{1}{1-\alpha}\right) \frac{\sigma_\mu}{\sigma_p}$. In contrast to the i.i.d. case, all three moments are now necessary to infer the extent of uncertainty. Otherwise, the intuition is very similar: all else equal, a higher relative correlation ($\rho_{pk} - \rho_{pa}$) implies greater uncertainty, and a lower $\rho_{pa}$, higher levels of noise in prices.

The identification equations in (29) hold in the absence of other distortions. Importantly, however, we show in Appendix I.F that the presence of other distortions affects our inference in the permanent shocks case exactly as in the i.i.d. case - correlated distortions do not bias our measure of uncertainty while uncorrelated distortions lead us to a lower bound on the extent of uncertainty.

**III.B. Robustness**

We now turn to two additional exercises aimed at further demonstrating the robustness of our identification strategy. In the first, we generalize the information structure to allow for arbitrary correlations between signal errors. In the second, we examine the implications of measurement error. For simplicity, we abstract from other distortions (i.e., set $\tau_{it} = 0$). Here, we discuss only the i.i.d. case (the Appendix repeats the analysis for permanent shocks).

**Alternative information structures.** In our baseline setup, the noise terms in the signals received by firms and investors are assumed to be orthogonal to each other. This may be an unrealistic assumption - in practice, firms routinely release forecasts and announce their investment plans to investors/analysts. This could induce correlation in the signal errors (specifically, $e_{it}$, $v_{ijt}$, and $z_{it}$), raising a potential concern - this is a source of co-movement between investment and returns and so may bias the inference of $\mathbb{V}$. However, this turns out not to be the case - the relative correlation still leads us to the true $\mathbb{V}$. More precisely, (26) holds for an arbitrary correlation structure between market and firm information.\(^{35}\) Therefore, our assessment of firm-level uncertainty is not sensitive to assumptions about the structure of information flows between firms and investors. Note that this only applies to our estimate of $\mathbb{V}$ - our conclusions about the individual variances ($\sigma^2_c, \sigma^2_v$ and $\sigma^2_z$) do change with assumptions about correlations.

\(^{35}\) See Appendix I.G.
Measurement error. Another important concern is measurement error. In this section, we consider, separately, the effects of classical measurement in observed revenues and capital.

We turn first to revenues: suppose observed revenues are given by

$$\hat{rev}_{it} = rev_{it} + \epsilon^y_{it} \quad \epsilon^y_{it} \sim N(0, \sigma^2_{\epsilon_y})$$

The error term $\epsilon^y_{it}$ inflates the variance of measured fundamentals, $\sigma^2_{\mu}$, and attenuates their observed correlation with returns, $\rho_{pa}$. Since $\rho_{pk}$ is unaffected, this leads to an upward bias in measured uncertainty.

Formally, using the observed correlations (instead of the true ones) in (26) leads us to measure uncertainty as

$$1 - \frac{\rho^2_{pa}}{\rho^2_{pk}} \left( \frac{\sigma^2_{\mu}}{\sigma^2_{\mu} + \sigma^2_{\epsilon_y}} \right) > 1 - \frac{\sigma^2_{\epsilon_y}}{\sigma^2_{\mu}} \iff \frac{\sigma^2_{\epsilon_y}}{\sigma^2_{\mu}} < \frac{2\alpha - 1}{\alpha^2}$$

In Section IV, we use our general model to quantitatively assess the extent of this bias. We find that, for reasonable values of $\sigma^2_{\epsilon_y}$, it is relatively modest, particularly for China and India.

We turn next to capital, which presumably is even more vulnerable to mismeasurement. Suppose the observed capital input is

$$\hat{k}_{it} = k_{it} + \epsilon^k_{it} \quad \epsilon^k_{it} \sim N(0, \sigma^2_{\epsilon_k})$$

In this case, the error term lowers the correlations of fundamentals with returns (as before) and now additionally, investment with returns, so the net effect on the relative correlation and therefore on $\frac{\rho^2_{pa}}{\rho^2_{pk}}$ is ambiguous. We can show the net effect is positive (i.e., we overstate $\frac{\rho^2_{pa}}{\rho^2_{pk}}$) when the true degree of uncertainty is sufficiently low:

$$1 - \frac{\rho^2_{pa}}{\rho^2_{pk}} \left( \frac{\sigma^2_{\mu}}{\sigma^2_{\mu} + (1 - \alpha^2)\sigma^2_{\epsilon_k}} \right) > 1 - \frac{\sigma^2_{\epsilon_k}}{\sigma^2_{\mu}} \iff \frac{\sigma^2_{\epsilon_k}}{\sigma^2_{\mu}} < \frac{2\alpha - 1}{\alpha^2}$$

Numerical simulations in Section IV generally point to a negative bias, i.e., to the extent that capital is measured with error, our estimates of uncertainty are conservative.
IV. QUANTITATIVE ANALYSIS

Our analytic results in the previous section demonstrated a tight relationship between the moments \( (\rho_{pk}, \rho_{pa}, \sigma_p^2) \) and the informational parameters \( (\sigma_e^2, \sigma_v^2, \sigma_z^2) \), and through them, the degree of micro-level uncertainty. Moreover, the results there suggest that our empirical strategy leads us to an accurate, or even conservative, estimate of uncertainty without requiring us to explicitly account for and quantify the host of other factors that may distort input choices. With this intuition in hand, we now return to our general model and use an identification approach quite similar in spirit to infer these parameters using data from 3 countries - the US, China, and India. We use the results to quantify the extent of informational frictions, the degree of resulting misallocation, and the impact on aggregate outcomes in all 3 countries. Our analysis also sheds light on the role of learning from various channels, including the role of financial markets in improving allocative efficiency by delivering useful information to firms. We also make use of the multi-country aspect of our analysis to perform a number of counterfactual experiments assessing the sources of cross-country variation in uncertainty and their respective effects on allocative efficiency.

IV.A. Parameterization

We begin by assigning values to the more standard parameters in our model - specifically, those governing the preference and production structure of the economy. Throughout our analysis, we will hold these constant across countries; cross-country differences will come only from the parameters governing the stochastic processes on firm fundamentals and learning. Although a simplification, we feel that this is a natural starting point and allows us to focus on the role of imperfect information in leading to differences in aggregate outcomes across countries.

An important issue here is the choice of period length in the model. The focus of this paper - investment decisions - and our specific modeling choices push us towards larger period lengths. There is significant empirical evidence of long lags in planning and implementing investment projects, with estimates of the mean duration time between the planning stage and project completion of between 2 and 3 years.\(^\text{39}\) It seems reasonable then to assume that firms are required to forecast fundamentals over a relatively long horizon when making large investment decisions and have only limited flexibility to adjust capital choices ex-post in response to additional information. One approach would be to explicitly model these lags as well as

\(^{39}\) The classic reference here is Mayer (1960), who uses survey data on new industrial plants and additions to existing plants to find a mean gestation lag between the drawing of plans and the completion of construction of about 22 months. More recently, Koeva (2000) finds the average length of time-to-build lags to be about 2 years in most industries, defined as the period between the announcement of new construction and the ensuing date of completion. Given that this excludes the planning period prior to the announcement date, the total gestation lag is likely somewhat longer.
other features (such as adjustment costs/irreversibilities) that are likely relevant for investment decisions over shorter horizons. Largely for tractability, we take a different route and model the capital choice as a static one, but interpret a period in the model as spanning a relatively long horizon, specifically 3 years. This makes the omission of explicit lags/irreversibilities, etc. somewhat less of a concern, while, importantly, allowing us to preserve the simple expressions linking uncertainty to aggregate outcomes.\(^{40}\)

In line with our choice of period length, we set the discount rate \(\beta\) equal to 0.9. We assume constant returns to scale in production and set the production parameters \(\hat{\alpha}_1\) and \(\hat{\alpha}_2\) to standard values of 0.33 and 0.67, respectively. Given our CRS assumption, firm scale is then limited only by the curvature in demand, captured by the elasticity of substitution \(\theta\). This will be a key parameter in influencing the quantitative impact of informational frictions. The literature contains a wide range of estimates for this parameter. We set our baseline value at 6, roughly in the middle of the commonly used range.\(^{41}\) Our choice of \(\theta\) translates into an \(\hat{\alpha} = 0.62\) in case 2 (only capital is chosen under imperfect information) and \(\alpha = 0.83\) in case 1 (both inputs are chosen under imperfect information).

Next, we turn to the country-specific parameters. We begin with those governing the process on firm fundamentals \(a_{it}\): the persistence \(\rho\) and variance of the innovations \(\sigma^2_{\mu}\). In both of our cases, we can directly construct the fundamental for each firm (up to an additive constant) as \(va_{it} - \alpha k_{it}\), where \(va_{it}\) denotes the log of value-added, and the value of \(\alpha\) depends on whether we are in case 1 or case 2.\(^{42}\) We then estimate the parameters of the fundamental process by performing the autoregression implied by (1), additionally including a time fixed-effect in order to isolate the idiosyncratic component of the innovations. The resulting coefficient delivers an estimate for \(\rho\) and the variance of the residuals for \(\sigma^2_{\mu}\).\(^{43}\)

Finally, it remains to pin down the three informational parameters, i.e., the variances of the error terms in firm and investor signals \(\sigma^2_e\) and \(\sigma^2_{\nu}\), and the variance of the noise trader shock \(\sigma^2_z\). We follow almost directly the strategy outlined above, i.e., target second moments in the growth rates of firm-level investment, fundamentals, and prices.\(^{44}\) The precise moments we use are the cross-sectional correlations of stock returns (denoted \(\rho_{\mu}\)) and changes in fundamentals, as well the variance of returns (as above, denoted \(\rho_{\nu}\) and \(\sigma^2_p\), respectively). Importantly, stock returns are lagged by one period, so that the correlations reflect the comovement of investment and fundamentals with stock returns over the preceding

\(^{40}\) Morck, Shleifer, Vishny, Shapiro, and Poterba (1990) make a similar argument and perform their baseline empirical analysis using 3-year spans. They also point out that the explanatory power of investment growth regressions at shorter horizons (e.g., 1 year) are quite low.

\(^{41}\) Additionally, we report our results, at least in case 2, for two other values of \(\theta\), namely 4 and 10.

\(^{42}\) Substitute either (4) or (7) into (2).

\(^{43}\) See Appendix for details.

\(^{44}\) We follow the literature examining the feedback effect of stock prices and use growth rates of investment (in the analytical cases studied earlier, we used investment, i.e. the growth rate of capital). By working with growth rates, we partly cleanse the data of firm fixed-effects, which are a significant component in cross-sectional differences in these variables. See Morck, Shleifer, Vishny, Shapiro, and Poterba (1990) for a more detailed discussion of these issues.
period. This avoids feedback from investment and fundamentals to returns, the reverse of the relationship in which we are interested. Table I summarizes our empirical approach.

[Table I about here.]

We use a simulated method of moments (SMM) approach to assign values to the informational parameters. Formally, we search over the parameter vector \( \left( \sigma_e^2, \sigma_v^2, \sigma_z^2 \right) \) to find the combination that minimizes the (unweighted) sum of squared deviations of the model-implied moments from the target moments. Before reporting the parameter estimates, we provide a numerical analogue of the identification argument in the previous section. This more general version of the model does not yield analytical expressions for the moments of interest, but we show in Figure I that the relationships between moments and parameters highlighted above go through almost exactly. The first panel shows a positive relationship between \( V \) and the relative correlation, as suggested by equations (26) and (29). Similarly, in the second panel, we see that higher levels of noise in prices are associated with lower \( \rho_{pa} \), exactly as we saw in the analytical cases. Finally, the bottom panel shows that, holding fixed the total noise \( \left( \sigma_v^2 \sigma_z^2 \right) \), increasing the size of the noise trader shock makes returns more volatile.

[Figure I about here.]

**IV.B. Data and Parameter Values**

We construct the target moments using data on firm-level production variables and stock returns from Compustat North America for the US and Compustat Global for China and India. We focus on a single cross-section of firms in each country for the year 2012. This is the period with the largest number of observations, particularly so for China and India. Note that our data requirements are quite stringent: due to our focus on 3 year horizons and our use of growth rates, we require at least 9 consecutive years of data for a firm in order to construct two 3 year periods and include it in our sample, with 2012 representing the final year of the second period.

We compute the firm’s capital stock \( k_{it} \) in each period as the average of its property, plant and equipment (PP&E) over the relevant 3 years, and investment as the change in the capital stock relative to the preceding 3 year period. To construct our measure of the fundamental \( a_{it} \), we compute sales/revenue analogously as the average over the 3 year period and calculate \( a_{it} = va_{it} - \alpha k_{it} \). We compute value-added from revenues using a share of intermediates of 0.5. First-differencing gives investment and changes in fundamentals between

45. Here, we plot the difference in correlations, i.e. \( \rho_{pi} - \rho_{pa} \), but using the ratio yields a very similar picture. All moments in the figure are computed using simulated firm-level data from the model as we vary the corresponding parameter, holding the remaining parameters fixed at their estimated values (for the US).

46. Here, we hold \( V \) and the ratio \( \frac{\sigma_v}{\sigma_z} \) fixed, so as to focus on the overall informational content of prices.
the two periods. Stock returns are constructed as the change in the firm’s stock price over each 3 year period, adjusted for splits and dividend distributions. In order to be comparable to the unlevered returns in our model, stock returns in the data need to be adjusted for financial leverage. To do so, we assume that claims to firm profits are sold to investors in the form of debt and equity in a constant proportion (within each country). Observed return variances must then be multiplied by a constant factor in order to make them comparable to returns in the model, where the factor depends on the ratio of debt to total assets (or alternatively, debt to equity). Computing the debt-asset ratio from our sample gives average values of about 0.30 in both the US and India, and about 0.16 in China, with corresponding adjustment factors of about 0.5 and 0.7, respectively. All values reported below reflect this adjustment.\textsuperscript{47} To isolate the firm-specific variation in our data series, we extract a time fixed-effect from each and utilize the residual as the component that is idiosyncratic to the firm. This is equivalent to demeaning each series from the unweighted average in each time period and serves to eliminate any aggregate component, i.e., changes in aggregate conditions, for example, or inflation.\textsuperscript{48} As mentioned above, the target moments are computed using returns lagged by one period.\textsuperscript{49} This avoids the simultaneity problems associated with comparing price movements to contemporaneous investment/fundamentals numbers. We trim the 2\% tails of each series in order to eliminate outliers. Appendix provides further details on how we build our sample and construct the variables.

We report the three target moments in the left hand panel of Table II both for case 2 in the upper panel and case 1 in the lower.\textsuperscript{50} In both cases, the moments exhibit significant cross-country variation. The US and India show similar levels of return volatility and a similar relation between returns and investment growth, but quite different comovements with fundamentals - returns in the US are more highly correlated with future changes in fundamentals. China has a return variance that is almost half that of the other two countries, along with the lowest correlations of returns with investment and fundamentals. As made clear by the analytic results in Section III, none of these moments is a sufficient statistic to identify the informational role of markets or infer the extent of micro-level uncertainty; rather, it is the joint pattern of the three moments that matters and our explicit modeling of both production and financial market activity is precisely what allows us to tease this out.

In the right hand panel of Table II, we report the resulting parameter values. The parameters governing the process on fundamentals $\rho$ and $\sigma_\mu$ are inferred from the regression implied by (1) as detailed above and

\textsuperscript{47} In brief, letting $d$ denote the debt-asset ratio, the observed return variances must be multiplied by $(1 - d)^2$ to obtain the variance of unlevered returns. We describe the details of the calculations in Appendix.

\textsuperscript{48} An alternative is to use CAPM $\beta$’s to remove the aggregate component from individual stock returns. This approach yields very similar results.

\textsuperscript{49} For example, $\rho_{pi}$ is the correlation between 2006-09 returns and investment growth during 2009-12.

\textsuperscript{50} $\rho_{pa}$ changes across cases since $\alpha$ affects our estimates of $\alpha$. The remaining two moments change almost imperceptibly due to trimming.
the informational parameters from the target moments and SMM procedure just described. As we would expect from the cross-country variation in the target moments, the parameter estimates also vary markedly across countries. The US has less volatile fundamental shocks and lower levels of noise both in private signals at the firm level (lower $\sigma_e$) and generally in the stock market as well (lower $\sigma_v$ and $\sigma_z$). In the next section, we gauge the detrimental impact of the estimated frictions on productivity and output and in leading to differences in these aggregates across countries.

[Table II about here.]

**IV.C. Results**

We report our baseline results in Table III. The first two columns present the implied value for $V$ based on the parameter estimates in Table II, both in absolute terms and as a percentage of the underlying fundamental uncertainty, $\sigma_\mu^2$. The next two columns show the degree of misallocation in our sample as measured by the total dispersion in the MRPK, $\sigma_{\text{mrpk}}^2$, and the share that informational frictions account for.\(^5\) In the last two columns, we compute the implied losses in aggregate TFP and output relative to a full information benchmark. The top panel contains results for case 2, in which only capital is chosen under imperfect information, and the bottom panel the analogous results for case 1, in which both capital and labor are. Case 2 is the more conservative scenario (in the sense that it leads to lower TFP/output losses). We return to this issue in our discussion below and provide some suggestive evidence that reality likely falls in between the two cases.

[Table III about here.]

Turning to the first two columns, there is substantial uncertainty in all cases: as a percent of the fundamental uncertainty $\sigma_\mu^2$, the residual uncertainty ranges from a low of 41% in the US to a high of 77% in India in case 2, and similarly in case 1, although China and the US move much closer together on this score. In other words, firm learning ($\sigma_\mu^2 - V$) eliminates from a high of about 60% of total uncertainty in the US to a low of about 20% in India in case 2, with a generally similar pattern in case 1, although the estimated degree of learning falls in the US and India. As we would expect from the cross-country variation in the parameter estimates in Table II, the level of uncertainty varies systematically across countries: US firms are the most informed and Indian firms the least, with Chinese firms falling in the middle.

Next, we ask, how much of total MRPK dispersion do informational frictions account for? The answer is a significant portion: as a percentage of the total $\sigma_{\text{mrpk}}^2$, $V$ represents between about 20% and almost

---

\(^5\) $\sigma_{\text{mrpk}}^2$ is computed as the average of within-industry dispersion in each country.
60%, with the share generally lower in the US than in the two emerging markets. Later, we will decompose the observed MRPK into a firm fixed effect and a transitory component and assess the role of informational frictions in explaining the latter. In a sense, this is a more meaningful comparison because informational frictions really cannot speak to fixed effects, which seem to an important component of observed MRPK dispersion.

Finally, the last two columns of Table III show that the substantial degree of residual uncertainty implies significant losses in productivity and output. Compared to the full-information benchmark, in case 2, losses in steady state TFP range from 4% in the US (more precisely, 0.04 log-points) to 10% in India, with corresponding output losses of 5% to 14%.\textsuperscript{52} Estimated losses are significantly higher in case 1, ranging from 40% to about 80% in productivity and 60% to over 1 in output. In both cases, the US exhibits the smallest losses, reflecting the fact that US firms exhibit the smallest degree of ex-post uncertainty, and India the largest, with China falling in the middle. The differential impact of informational frictions leads to significant differences between the US and the two emerging markets in productivity and output, ranging from 3% to almost 40% for the former (across the two cases) and from 5% to over 50% for the latter (subtract $a^* - a$ and $y^* - y$ for the US from the corresponding values for China and India). These are somewhat modest, however, in comparison to standard measures of cross-country differences in aggregate TFP and income per-capita, but very much in line with other estimates based on particular frictions - for example, Midrigan and Xu (2013). In Section IV.E, we will use the structural parameters of our model to investigate in more detail the differences in informational frictions across countries; specifically, we perform a number of counterfactual experiments exploring the potential gains in the emerging markets from having access to a US-quality information structure.

\textbf{Case 1 vs Case 2.} Table III shows that under either scenario, informational frictions have a significant detrimental impact on aggregate performance, both when comparing the parameterized economies to their full information benchmarks and when comparing the consequences of these frictions in emerging markets to the US. The magnitude of the effects depend to a great extent on the nature of the firm’s input decisions, that is, whether all inputs are chosen subject to the friction or only traditionally quasi-fixed inputs like capital. The intuition for these differences comes directly from equations (11) and (12): in case 1, given the level of uncertainty $V$, the aggregate consequences depend only on the elasticity of substitution $\theta$; in case 2, the relative shares of capital and labor in production also matter. The greater is labor’s share $\hat{\alpha}_2$, the greater the ability of firms to mitigate the effects of capital misallocation from the informational friction by reallocating labor in a compensating fashion. Any such flexibility is absent in case 1, in which both inputs

\textsuperscript{52} In what follows, we adopt the convention of referring to log points as percentages.
are subject to the same friction. Thus, the aggregate “cost” of a given level of uncertainty is larger in case 1 than case 2.

It seems reasonable to conjecture that reality lies in between these two extreme cases, i.e., that firms make labor decisions under some uncertainty, but that this is likely to be less than what they face when making investment decisions. In this sense, these two cases provide bounds on the adverse consequences of imperfect information for aggregate performance. With detailed data on labor inputs, one could use a model with a more flexible information structure and obtain a sharper estimate. Unfortunately, even for the US firms in our sample, such data is hard to come by.\textsuperscript{53}

Despite this limitation, we use total employment as an admittedly rough measure of labor input to perform two illustrative calculations for US firms. In the first, we re-estimate the informational parameters using moments in labor (instead of capital). This yields an estimate of residual uncertainty at the time of the labor choice. We express this as a fraction of the prior variance and denote it by \( \left( \frac{\nu}{\sigma^2} \right)_{\text{Lab}} \). If reality were well described by case 1, then \( \left( \frac{\nu}{\sigma^2} \right)_{\text{Lab}} \) should be quite close to the \( \frac{\nu}{\sigma^2} \) estimated using capital data under case 1; if reality were closer to case 2, then we should recover little or no residual uncertainty. Thus, comparing these two estimates is an indication of where we lie between the two cases. Performing this calculation for US firms leads to an estimate of \( \left( \frac{\nu}{\sigma^2} \right)_{\text{Lab}} = 0.33 \).\textsuperscript{54} 

Comparing this to the \( \frac{\nu}{\sigma^2} \) estimate of 0.63 in Table III suggests that the aggregate effects of uncertainty in the US is roughly halfway between the case 1 and case 2 figures in that table.

Our second exercise compares the observed dispersion in the marginal revenue product of labor (MRPL) to that in the MRPK. Note from equations (7) and (6) that in case 2, the marginal revenue product of labor (MRPL) is equalized across firms so that there should be no dispersion, while in case 1, dispersion in the MRPL exactly equals that in the MRPK.\textsuperscript{55} This follows directly from our assumption that in the former, all information is revealed before the labor choice, while in the latter, labor is chosen under the same limited information as capital. This implies a link between the cases and the ratio of the dispersion in the MRPL to that in the MRPK, i.e., \( \frac{\sigma^2_{\text{MRPL}}}{\sigma^2_{\text{MRPK}}} = 0 \) in case 2 and = 1 in case 1. We compute this ratio for the US firms in our sample.\textsuperscript{56} Using total employment as the measure of labor input, the ratio is equal to 0.57; using data on the wage bill for the small set of firms for which it is available gives a similar value of 0.53. These results again suggest that reality is indeed in the middle of the two extremes we have analyzed. Certainly,

\textsuperscript{53} Data on number of employees is generally available for US firms. However, the presence of heterogeneity in human capital, hours worked, etc., make this a problematic measure of effective labor input. The misallocation literature typically uses the wage bill as a rough adjustment for these concerns. In our sample, data on the wage bill is available only for a small set of firms. For China and India, even employment data is reported only by a handful of firms.

\textsuperscript{54} Moments and parameters are reported in Appendix III.A.

\textsuperscript{55} Note, however, that these statements hold with equality only in the absence of other distortions that disproportionately affect the use of one input vs the other.

\textsuperscript{56} \( \sigma^2_{\text{MRPL}} \) is computed analogously to \( \sigma^2_{\text{MRPK}} \).
an important direction for future work is a more in-depth analysis of the precise nature of the firm’s input
choices.

**Transitory MRPK dispersion.** Table III shows that uncertainty can account for a significant portion
of the total MRPK dispersion observed in the data, ranging from 20-60% across countries and cases. Note,
however, that the dispersion induced by uncertainty is short-lived. In the data, on the other hand, there
is some evidence of permanent deviations - in other words, measured MRPK deviations seem to comprise
both a firm-specific fixed effect (which we cannot speak to with our theory) and a transitory component. In
a sense, a more appropriate gauge of the contribution of informational frictions would be a comparison to
this latter piece. To do so, we separate the two components for the US firms in our sample. We restrict
the sample to firms with at least 15 years of data and extend the data as far back as 1985, so that we can
construct at least 5 and as many as 10 3-year observations for each firm. We compute the MRPK for each
firm in each period and regress the result on a firm fixed-effect. The residuals from this regression capture
the purely transitory component of MRPK deviations. We then compute the variance of this object, i.e.,
the dispersion in the transitory component of MRPK deviations, and ask how much of this dispersion do
informational frictions account for? This exercise reveals, first, that dispersion in the transitory component
is substantial, representing about one-third of the total. Moreover, our estimated $V$ for the US accounts for
about 60% of the transitory dispersion in case 2, and just about the entirety in case 1, again pointing to the
empirical relevance of informational frictions.

**The effect of curvature.** The impact of informational frictions is sensitive to the degree of curvature,
captured here by the elasticity of substitution $\theta$, set equal to 6 in our baseline computations. Table IV
reports results for case 2 under two alternative values: $\theta = 4$, which is on the low end of the commonly
used range, and $\theta = 10$, which is on the high end. Changes in $\theta$ lead to significant changes in the effects of
the friction. Both the estimates of $V$ and their impact on TFP are lower with a smaller $\theta$, with TFP losses
ranging from 2% in the US to 6% in India and higher, but similarly ordered, output losses. In contrast, the
higher value of $\theta$ shows a more severe aggregate impact, ranging from 8% in the US to 16% in India for TFP
and from 11% to almost 25% for output. The impact of the friction varies with $\theta$ for two reasons: first, as
can be seen in (12), for a given $V$ the aggregate consequences are larger for higher values of $\theta$ (i.e., higher
$\alpha$). Second, the estimated $V$ itself increases with $\theta$, due to the fact that the $\rho_{pa}$ we measure from the data

57. Sufficient data to perform this analysis are not available in the other two countries.
58. We again compute the average of within-industry dispersion.
59. The sensitivity of our results to $\theta$ is not particular to our framework, but rather, is common when using this class of
model to study the aggregate implications of misallocation. See, for example, Hsieh and Klenow (2009), who find that gains
from marginal product equalization approximately double in China and India when moving from $\theta = 3$ to $\theta = 5.$
falls as $\theta$ increases but the other two moments (importantly, $\rho_{pi}$) are unchanged.

[Table IV about here.]

### IV.D. The Sources of Learning

**Decomposing $V$.** Table III shows that firm learning can be quite significant and potentially mitigates a substantial portion of the underlying fundamental uncertainty. Here, we explore the relative importance of the two sources of learning present in our model, i.e., private sources within the firm versus market prices. We begin by reporting in the left-hand panel of Table V the total extent of learning and its aggregate consequences. To do so, we compute the reduction in $V$ both in absolute and percentage terms due to learning from both channels, i.e., $\Delta V = V - \sigma_\mu^2$, and the resulting effects on aggregate productivity and output. The table shows that total learning can be quite important and translates into significant improvements in TFP and output: in case 2 these range from 3% in India to 5% in the US for the former and from 4% to 8% for the latter, with substantially higher gains in case 1.60 Interestingly, the contribution of learning in China in case 1 appears comparable to that in the US, even though Table II showed higher levels of signal noise in China. This is simply due to the fact that Chinese firms start out with more underlying uncertainty (i.e., $\sigma_\mu^2$), and so even a less informative signal can be more valuable.

[Table V about here.]

To break down the sources of learning, it is easier to work with the inverse of $V$, i.e., the total precision of the firm’s information, which lends itself to a simple linear decomposition:

$$\frac{1}{V} = \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}$$

We focus on the latter two terms, which capture the contributions of private and market learning to overall precision and compute their relative import as $\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}$ and $\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}$, respectively, i.e., as the fraction of the total increase in precision due to each source. We report the results in the right hand panel of Table V. Strikingly, learning is due overwhelmingly to private sources: at best, markets account for about 10% of the increase in precision in the US and India, and about half of this in China. As we will see, this limited informational role of markets will prove to be a robust finding throughout our analysis.

**The role of market information.** To explore in greater depth the importance of new information in stock prices, we recompute $V$ under the assumption that firms learn nothing from these prices, i.e., that they

---

60. Note the difference between these calculations and those in Table III. There, we compute losses compared to a full-information benchmark. Here, we are computing gains versus a no-information (about innovations to fundamentals) benchmark.
contain no information. This simply entails sending the noise in markets, \( \sigma_\nu^2 \sigma_\varepsilon^2 \), to infinity. The change in \( V \) is a measure of the contribution of stock market information to firm learning, given by

\[
\Delta V = V - \left( \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_\varepsilon^2} \right)^{-1}
\]

We report this value in the first column of Table VI (as a percentage of the prior uncertainty \( \sigma_\mu^2 \)), along with the associated gain in aggregate TFP (for the sake of parsimony, we don’t show the gain in aggregate output, which is simply a constant multiple, 1.5, of the gain in TFP). Even for the US, which has the most informative prices, market information reduces uncertainty only modestly, eliminating 2% of the fundamental uncertainty, with associated TFP gains between 0.2% and 1.3%. For China and India, the contribution of market-produced information is even smaller, reducing uncertainty by between 0.5% and 1.9%, which represents a maximum gain in TFP of 0.8%.

Next, we ask whether the limited informational role of markets is due to the level of noise in prices or to the fact that firms already have considerable information about fundamentals, mitigating the incremental contribution of market information. This distinction is related to the concepts of forecasting price efficiency (FPE) versus revelatory price efficiency (RPE) as put forth in Bond, Edmans, and Goldstein (2012). The former captures the extent to which prices reflect and predict fundamentals, i.e., the absolute level of information in prices; the latter, the extent to which prices promote real efficiency by revealing new information to the firm. In our framework, \( \Delta V \) in the first columns of Table VI is the natural measure of RPE, since it is the marginal impact of the information in prices on uncertainty, given the other sources of firm information. As the table shows, at the estimated parameter values, markets in all 3 countries exhibit relatively low RPE. To measure FPE, we compute the reduction in \( V \) from the information in stock prices alone, i.e.,

\[
\Delta V = \left( \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_\nu^2 \sigma_\varepsilon^2} \right)^{-1} - \sigma_\mu^2
\]

We report the results in the second set of columns of Table VI. Even as the only source of learning, the information provided by stock markets leads to only modest reductions in uncertainty. For the US, market generated information reduces fundamental uncertainty by between 5% and 10%, with an associated TFP effect that is as low as 0.9% at highest 3%. The aggregate impact of markets is even lower in the two emerging economies than in the US. Even in a forecasting sense then, the efficiency of stock market prices is fairly low, suggesting that the limited role of market information is in large part due to its poor quality; in other words, the poor RPE of stock markets can largely be attributed to their low FPE.
While the limited informational role of markets is a robust pattern across countries and cases, these results should be interpreted somewhat cautiously. Our analysis focuses exclusively on information and decisions over the medium term. It is silent on the role of stock markets in guiding longer-term, strategic decisions (e.g., the decision to enter a new market or acquire another company). In fact, one possible source of ‘noise’ in stock returns is information about the firm’s prospects over a much longer horizon. To the extent that it is orthogonal to fundamentals over the medium term, the relevant object for the firm’s current decisions, our strategy will still lead us to the right measure of uncertainty and the associated misallocation - our estimates for the specific informational parameters and the informational role of financial markets, however, are likely to be sensitive to assumptions about the nature of this ‘noise’ term.

Our analysis also abstracts from the contribution of financial market information in mitigating uncertainty about aggregate (or industry) conditions, particularly for outsiders (e.g., potential entrants, creditors, regulators, etc.). Explicitly modeling these features in a fully-fledged general equilibrium environment is a challenging task, but may well be essential for a comprehensive evaluation of the informational role of well-functioning financial markets.61

The role of private information. Next, we explore the contribution of learning from information generated within the firm (i.e., firm ‘private information’) by performing experiments analogous to those above: specifically, we calculate the marginal contribution of private information to allocative efficiency, both in the presence of market information and when it is the only source of learning for firms.

We report the results in the last two sets of columns of Table VI. In contrast to the effect of market learning, firm private information plays a much larger role in reducing uncertainty and improving aggregate performance. Turning to the first set of columns, private information eliminates between about 30% and 50% of the fundamental uncertainty in the US. The associated TFP gains are substantial, ranging from 4-20%. The corresponding figures are smaller for China and India in case 2 (although only slightly so in China), reflecting a worse quality of firm private information in those countries, and are smaller for India in case 1. Having said that, they are still quite significant across countries and are noticeably larger than the effects of market information. Similar to what we saw in Table V, the contribution of private learning in China in case 1 is on par with that in US, again because even a less precise signal can have more value if the extent of uncertainty is greater.

The last columns in Table VI reports the contribution of firm private information were it the only source of learning. A comparison with the previous columns shows that the values are quite similar - the presence of market information does not significantly alter the importance of private information for overall learning.

61. See section 5 in Bond, Edmans, and Goldstein (2012) for a related discussion.
IV.E. Cross-Country Counterfactuals

Next, we use our estimates to perform a number of instructive counterfactual experiments. Specifically, we assess the potential gains to China and India from having access to US quality information, whether through more informative financial markets or from better firm-level private information. In the first experiment, we compute the change in $V$ and associated aggregate gains in China and India under the assumption that the informativeness of financial markets - summarized by $\sigma_\nu^2\sigma_\sigma^2$ - is equal to that in the US, leaving all other country-specific parameters fixed. Second, we perform the same exercise for firm private information, that is, compute the change in $V$ and aggregate improvements under the assumption that firms in China and India have the same $\sigma_e^2$ as their US counterparts (again leaving the other country-specific parameters fixed). In a final experiment, we turn away from learning and study the role of fundamentals in leading to a differential impact of informational frictions across countries; to do so, we recompute $V$ in China and India assuming that firms in these countries face the same fundamental shocks as do US firms.

We report the results from these experiments in Table VII. The top panel shows that delivering US-quality markets to the emerging economies results in potentially significant yet modest reductions in uncertainty. As a percent of total fundamental uncertainty, these range from 2% to 7% (across cases) with a corresponding aggregate impact ranging, for example, from 1% to 6% in output. The middle panel shows that access to US-quality private information would have a much larger impact, reducing $V$ by as much as 40% of the underlying uncertainty in the most optimistic case, and, excepting China in case 1 which is an outlier here, more than 25% in the other cases. The resulting aggregate gains can be substantial, ranging, for example, from 4% to almost 40% in output. Across the board, the gains from accessing US-quality private information are much larger than from US-quality market information. To the extent that differences in learning lead to cross-country variation in economic aggregates, these disparities appear to be largely due to a lack of high quality firm private information, rather than to a lack of well-functioning (in an informational sense) financial markets in emerging markets. This is yet another instance of a result we have now seen several times: financial markets play only a modest informational role in promoting aggregate efficiency.

Finally, we recompute $V$ in China and India under the assumption that firms in those countries face shocks of the same volatility as those in the US, i.e., we replace $\sigma_e^2$ in the two countries with that of the US. We report the results in the bottom panel of Table VII, which shows that disparities in the volatility of shocks also contribute to differences in the impact of informational frictions. Exposure to a fundamental

---

62. Asker, Collard-Wexler, and De Loecker (2012) also highlight the role of different firm-specific shock processes in generating misallocation in a model with capital adjustment costs.
process such as that in the US reduces $\mathcal{V}$ between about 10% and 25% of the underlying uncertainty, leading to TFP and output gains that are potentially substantial, ranging from 1-20% and 2-30% respectively.\footnote{These numbers correspond to $\Delta(a - a^*)$ and $\Delta(y - y^*)$. Recall that $a^*$, the full information productivity level, is also affected by the size of fundamental shocks, $\sigma_{\mu_0}^2$.}

\textbf{IV.F. Robustness}

We now turn to a number of important robustness exercises. In Section III, we used our special cases to demonstrate the validity of our measurement approach (1) in the presence of other factors entering the firm’s capital choice, (2) with a richer correlation structure between firm and market information, and (3) in the presence of measurement error. To explore the extent to which the intuition from that section carries over to the general model, in this section we conduct three numerical experiments in the same spirit. Specifically, we explore the effects of (1) an alternative information structure in which investors observe a noisy signal of the firm’s private signal (in contrast to the conditional independence of signals in the baseline model), (2) a specific friction in the firm’s capital choice, namely adjustment costs, and (3) measurement error in either observed revenues or capital. In these experiments, our goal is to assess how the presence of these alternative factors affects our measure of the severity of informational frictions; the results from each continue to demonstrate the robustness of our baseline results.

\textbf{Correlated information.} Recall that in the special cases analyzed in Section III, our identification strategy proved robust to an arbitrary correlation structure, i.e., no matter how rich or complex the flow of information between firms and markets, our measure of uncertainty remained valid. Here, the lack of an analytical characterization means that we have to resort to numerical experiments to demonstrate a similar robustness. Without direct data on firm/market information, it is not obvious how to discipline the extent of the commonality of information. We take a different route and show results from a rather extreme modification of our baseline information structure - we assume that the signal observed by investors is a noisy signal of the firm’s private signal, i.e.,

$$s_{ijt} = a_{it} + e_{it} + v_{ijt}$$

Investors then have 2 sources of information - imperfect readings of the firm’s own information and the stock price. Notice that by construction, firms have nothing to learn from market prices. Despite that, investment and returns will comove in part due to the common component in the two information sets. We estimate $\mathcal{V}$ in this modified version of the model using the same procedure and data as above. We report the results in Table VIII. The estimates for firm-level uncertainty are very close to those in our base case (which, of course, implies that the associated aggregate implications are also virtually unchanged). Thus, we
conclude that our estimates of uncertainty are not particularly sensitive to assumptions about correlation in firm and market information.64

[Table VIII about here.]

**Capital adjustment costs.** As we showed in Section III, our methodology is robust to the presence of other factors or distortions. The analysis in that section, while reassuring, was done with a stylized representation of these factors, primarily for analytical tractability. To explore this result in the general model, we ask how the presence of a particular factor, capital adjustment costs, affects our assessment of the severity of informational frictions.65 We focus on a polar case - an economy where fully informed firms are subject to convex costs of adjustment. We then take our model with informational frictions to moments generated from this hypothetical economy - our goal is to see whether our empirical strategy will lead us to an incorrect inference about uncertainty.

This experiment requires two modifications to our baseline model. First, following Bloom (2009), Cooper and Haltiwanger (2006), and Asker, Collard-Wexler, and De Loecker (2012), we subject firms to quadratic costs of adjusting capital $\zeta K_{t-1} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^2$.66 Second, we assume that firms have full information ($\sigma_e^2 = 0$) but that stock markets are noisy ($\sigma_v^2 \neq 0, \sigma_z^2 \neq 0$). We then parameterize and solve the model in general equilibrium.67 We choose values of $(\sigma_v^2, \sigma_z^2, \zeta)$ to match 3 moments in the data - the volatility of returns and their correlation with fundamentals, $\sigma_p^2$ and $\rho_{pa}$, and the cross-sectional variation in firm-level investment. The first two were also used in our baseline analysis to measure the information in prices; the last is added to discipline the adjustment cost parameter $\zeta$.68 We simulate data using this parameterized model and compute the correlation of prices with investment (growth), $\rho_{pi}$. Since $\rho_{pa}$ is directly targeted, the model-implied $\rho_{pi}$ reveals the extent to which adjustment costs generate a large relative correlation, the statistic at the heart of our inference about $\mathbb{V}$. Finally, we reestimate the parameters of our model with informational frictions.

64. Our estimates of the individual error variances do change, however.
65. We put particular focus on adjustment costs since Asker, Collard-Wexler, and De Loecker (2012) find that these types of costs can account for a significant portion of MRPK dispersion across industries and countries. The role of financial frictions in driving misallocation, on the other hand, is much clearer - Midrigan and Xu (2013), for example, find modest effects. Moreover, our analysis focuses on large publicly traded firms for whom financial constraints are less likely to be binding. Having said that, we performed two additional exercises to investigate the robustness of our results to the presence of financial frictions: first, we parameterized the model using moments from only the largest firms (top quartile by sales), who are presumably less susceptible to this type of friction. In the second exercise, we restricted attention to firms that did not raise substantial additional capital, i.e., where favorable changes in share prices did not seem to loosen financial constraints. For both of these subgroups, the results are similar to our baseline.
66. We focus on convex costs and abstract from the fixed-type costs also considered in the literature, as the former are most capable of delivering a configuration of moments that resemble those under informational frictions. Intuitively, convex costs imply that investment decisions respond to innovations in fundamentals only in part contemporaneously and then with a lag. This latter feature resembles the pattern implied by informational frictions.
67. Since the firm’s capital choice problem is now a dynamic one, we can no longer analytically characterize the joint distribution of fundamentals and capital across firms, but rather, solve numerically for this distribution (and the associated general equilibrium implications) in steady state. We refer the reader to Appendix III.B for details.
68. This is along the lines of the estimation strategy employed in Bloom (2009), Cooper and Haltiwanger (2006), and Asker, Collard-Wexler, and De Loecker (2012).
using this simulated value for $\rho_{pi}$, along with the other two moments (namely, $\sigma_p^2$ and $\rho_{pa}$).

We report the results in Table IX. The correlation of returns with investment choices implied by the full-information adjustment cost model is significantly lower than that observed in the data, leading to the substantially lower values for the relative correlation reported in the table. In other words, in the absence of firm-level uncertainty, investment decisions covary much less with stock returns even in the presence of adjustment costs. The relative correlation numbers suggest that applying our empirical strategy to these simulated moments would result in substantially lower estimates of $V$. The third column of the table confirms that this is indeed the case: the estimated $V$’s from the simulated data are only about one-third of those in our baseline analysis (compare to the fifth column). The resulting aggregate effects are also substantially lower: for example, TFP losses are only between 1 and 4%, compared to 4-10% in our baseline. These results provide some reassurance that our estimates are not simply picking up the effects of adjustment costs, but are indeed robust measures of firm-level uncertainty.

While this analysis had the admittedly limited goal of confirming the robustness of our estimates of uncertainty, it also suggests that both types of frictions (adjustment costs and imperfect information) might be necessary to match a broader set of empirical moments. Such a unified framework would also allow us to disentangle their effects on aggregate outcomes as well as explore how they interact with each other. At some level, it is not even clear that they should be thought of as two independent frictions: imperfect information leads to delayed responses to fundamentals and in this sense, provide a deeper theory of adjustment costs. Exploring these issues, both theoretically and quantitatively, is an important, if challenging, direction for future research.

**Measurement error.** Finally, in our last robustness exercise, we assess the extent to which our quantitative results are affected by classical measurement error. As before, we examine, separately, the effect of errors in observed revenues and capital. In each instance, we assume a size of the error, correct the relevant empirical moments accordingly, and re-parameterize the model to match these adjusted moments.

We turn first to measurement error in revenues. Suppose observed revenues are given by

$$\hat{rev}_{it} = rev_{it} + \epsilon_{yt}$$

$\epsilon_{yt} \sim N(0, \sigma_{\epsilon y}^2)$

69. We show in Appendix III.B the full set of target moments and resulting parameter values.
70. Fuchs, Green, and Papanikolaou (2013) provide a microfoundation for adjustment costs based on asymmetric information and adverse selection.
71. The expressions for the corrections to the moments are given in Appendix III.C, along with the moments and parameters for the cases we analyze.
We consider two levels of $\sigma^2_{\epsilon_y}$: namely, 10 and 25 percent of the observed cross-sectional dispersion in revenue growth in each country.\footnote{This is equivalent to assuming that 20% and 50%, respectively, of the observed dispersion in cross-sectional revenue growth is due to measurement error.} The middle two panels of Table X present the resulting estimates of $V$ and $\frac{V}{\sigma^2_{\rho \sigma}}$ for these two cases, along with our baseline estimates for comparison. They show that measurement error in revenues causes an upward bias in our uncertainty estimates. The intuition is the same as the special cases in Section III - measurement error in $\text{rev}_{it}$ attenuates $\rho_{pa}$, but does not affect the correlation of returns with investment, leading to a larger estimate for uncertainty. However, the bias seems to be economically significant only for the US. To put it differently, our conclusions about the severity of informational frictions in India and China - whether in absolute terms or, perhaps more importantly, in comparison to the US - appear to be robust to the presence of this type of error.\footnote{A potential concern with this exercise is that measurement error could be proportionately larger in the Indian and Chinese data. One way to account for this is by comparing across panels in Table X, i.e., by comparing the baseline estimates for the US in the first panel to those for India and China in the other panels. Such an exercise attenuates cross-country differences in the severity of informational frictions, but they remain economically meaningful.} 

Turning next to measurement error in capital, suppose instead that observed capital input is given by, i.e.,

$$\hat{k}_{it} = k_{it} + \epsilon_{it}^k \quad \epsilon_{it}^k \sim N(0, \sigma^2_{\epsilon_k})$$

We set $\sigma^2_{\epsilon_k}$ to equal 10 percent of the observed cross-sectional dispersion of revenue growth in each country.\footnote{Note that the observed variation in investment growth imposes an upper bound on the measurement error. Setting $\sigma^2_{\epsilon_k} = 0.25\sigma^2_{\Delta \text{rev}}$, for example, turns out to violate that bound for some countries/cases - the variation coming from the error term alone is greater than the observed variance in investment growth. In light of this (along with the fact that measurement error in $k_{it}$ seem to induce a downward bias in our estimates), we only show results for $\sigma^2_{\epsilon_k} = 0.10\sigma^2_{\Delta \text{rev}}$.} The right hand panel of Table X presents the results. They show that measurement error in $k_{it}$ causes us to underestimate the true degree of uncertainty. This is consistent with our findings in Section III, where we showed that measurement error in $k_{it}$ lowers both correlations and therefore, has ambiguous effects on our estimates of uncertainty. To the extent that mismeasurement is a more serious concern for capital than revenues, these results suggest that, if anything, measurement error may be biasing our estimates downward.

\section*{V. Further Evidence of the Mechanism}

In this section, we provide additional evidence that what we are measuring is indeed uncertainty at the micro level. Finding such corroborating evidence is not an easy task - as we have emphasized throughout this paper, directly measuring information is not straightforward. Indeed, it is precisely this difficulty that motivates the structural approach we take. With that caveat in mind, we compare our estimates of
uncertainty in the US at a more disaggregated level - specifically, across sectors and over time - to alternative indicators that are constructed through completely different approaches. We find that the patterns in our estimates of $\mathcal{V}$ line up well with these other indicators.

### V.A. Uncertainty Across Sectors

A priori, it is not clear which sectors of the economy face high micro-level uncertainty and which low, and so it is not obvious how to identify cross-sector variation in an attempt to validate our mechanism. To do so, we construct a novel, direct measure of firm-level uncertainty at a sectoral (2-digit SIC group) level. We obtain detailed data on revenue growth forecasts issued by firms from the I/B/E/S Guidance database. I/B/E/S Guidance collects companies’ own forecasts of future performance as delivered, for example, in earnings conference calls. We match these forecasts with actual realizations to compute forecast errors. Our direct indicator of uncertainty at the sectoral level is simply the mean squared forecast error within each 2-digit SIC group.

We then estimate $\mathcal{V}$ using our structural model with moments computed by sector (at the same level of aggregation). We merge the I/B/E/S and Compustat data so that the set of firms is consistent across the two measures. Figure II plots the mean squared forecast error from I/B/E/S against the estimated $\mathcal{V}$. The figure shows a strong positive relationship between the two: in other words, sectors with a high variance in forecast errors are generally the same as those where our methodology points to a high $\mathcal{V}$. While the shorter horizons of the I/B/E/S forecasts make it difficult to directly compare magnitudes (the revenue forecasts we use are approximately for a 1 year horizon - data on longer-term forecasts are quite sparse), we note that the two measures are comparable along this dimension as well. If we assume, for example, that the variance of forecast errors scales linearly with time, multiplying the mean squared error by three gives values that are on average about 70% of our estimated $\mathcal{V}$’s. Given the differences in data and methodology, this is a remarkable degree of consistency.

[Figure II about here.]

---

75. I/B/E/S describes the data as information “directly from management about future expectations,” which maps quite nicely into our notion of uncertainty. To the best of our knowledge, this is the first paper to use this data to measure micro uncertainty.

76. Appendix III.D contains details on our sample and construction of the indicator. A potential concern is that firms’ announced forecasts may contain strategic elements. These may affect our results to the extent they impact the variability of forecast errors systematically across sectors.

77. In fact, the relationship is monotonic with the exception of Wholesale Trade, which is a clear outlier in its $\mathcal{V}$. 

42
V.B. Uncertainty Through Time

Next, we explore the time series patterns in micro uncertainty implied by our methodology. To construct a time series of firm-level uncertainty, we compute the relevant moments and an estimate of $V$ year-by-year over the 15 year period spanning 1998-2012. To control for compositional and other low frequency changes, we use a balanced panel of firms and detrend our $V$ estimates using a simple linear trend. The top panel of Figure III plots the resulting series. It shows that uncertainty rises (relative to trend) during both of the recessions in our sample, a pattern that is consistent with the findings of the empirical literature aimed at measuring micro uncertainty. For example, Jurado, Ludvigson, and Ng (2013) use a factor-augmented VAR approach to isolate the unforecastable component of firm-level profits and define micro uncertainty as the variability of that component. This approach, like ours, seeks to distinguish volatility (i.e., the unconditional variability of fundamentals, $\sigma^2$ in our context) from uncertainty (i.e., the conditional variance, or $V$ in our language). The bottom panel of Figure III overlays the series from Jurado, Ludvigson, and Ng (2013) with our measure (their data is only available through 2011). The two series are broadly in line: both show significant increases in uncertainty, of roughly similar magnitudes, during both the 2000-01 and 2007-09 recessions.

In sum, these exercises show that our estimates of cross-sectional and time-series variation are broadly in line with other measures, constructed using very different data and approaches. These findings provide further reassurance as to the validity of our identification strategy and numerical results.

VI. Conclusion

In this paper, we have laid out a theory of informational frictions that distort the allocation of factors across heterogeneous firms, leading to reduced aggregate productivity and output. Taking this theory to the data, we find evidence of substantial micro-level uncertainty and associated aggregate losses, particularly so

---

78. As we go back further in time, the number of firms in our panel falls. Our choice of timing is geared towards having several data points prior to the 2001-02 cycle. Appendix III.E reports the full series of moments and parameters.

79. We use the term micro uncertainty to distinguish our estimates from indicators of uncertainty about aggregate conditions, or macro uncertainty. For example, following the seminal work of Bloom (2009), a widely used proxy for uncertainty is the VIX, which tracks implied volatility on the S&P 500 Index. This is also counter-cyclical, but, as a measure of macro uncertainty, is not directly comparable to our $V$.

80. We obtain the data from Sydney Ludvigson’s website at http://www.econ.nyu.edu/user/ludvigsons/. We take the longest forecast horizon available, 6 quarters, and use the average across quarters in each year.

81. Another proxy for micro uncertainty can be found in Bachmann, Elstner, and Sims (2013), who use forecast dispersion from the Philadelphia Fed’s Business Outlook Survey. Their series also displays a similar pattern over this time period.
in China and India.\textsuperscript{82}

There are several promising directions for future research. In our modeling approach, we have aimed to strike a balance between realism and transparency of the economic forces at play as well as of our empirical methodology. In doing so, we have made a couple of admittedly extreme assumptions. For example, the investment choice is modeled as a static and otherwise undistorted decision problem. Similarly, the learning process is rather stark, with perfect revelation at the end of each period, implying that firms are able to quickly ‘correct’ their past errors. As a result, investment in any given period includes a component related to the forecast error in the previous period. When shocks are highly persistent, this adjustment makes a substantial contribution to investment volatility. These assumptions limit our ability to match a larger set of micro moments on investment dynamics. Relaxing them is straightforward from a conceptual standpoint - for example, by introducing financial constraints, adjustment costs, or a richer stochastic process for fundamentals\textsuperscript{83} - however, there are substantial computational and empirical challenges. First, this introduces additional state variables into the firm’s problem and precludes the simple analytical mapping between uncertainty and aggregate outcomes. Second, this requires additional data to infer the additional parameters - such as the time series behavior of firm-level investment. This is less of an issue for the US, but in China and India, both availability and quality of the data decline dramatically as we go back in time.

Another important direction for future work is towards a deeper theory of differences in information quality. This is valuable both from a conceptual point of view as well as from the perspective of designing policies. For example, much of the cross-country variation we find seems to stem from differences in the quality of firms’ internal information sources and only marginally from the quality of information from financial markets. These results suggest that policies aimed at improving the quality of firm-level information may be more fruitful in improving aggregate performance than those intended to bring financial markets to a level closer to that in the US. Given that we follow the standard approach of modeling information as exogenous noisy signals, we are left to speculate on the exact form of such policies. In our view, the most likely sources of variation in the quality of firm private information are differences in manager skill and/or information collection/processing within the firm. Under this interpretation, improving manager training and/or accounting and information systems within firms holds the most promise of generating substantial gains in aggregate TFP. Our results also show that a reduction in fundamental volatility would also help mitigate the informational disadvantage of firms in less developed countries.

\textsuperscript{82} Strictly speaking, our sample covers only large, publicly traded firms. However, these are also likely to be the most well-informed. In this sense, one can view our estimates as a lower bound for the total effect of information frictions in the economy.

\textsuperscript{83} Suppose, for example, that the fundamental $a_{it}$ consists of a persistent component and a purely transitory one. Then, even if $a_{i,t-1}$ is observed perfectly ex-post, firms will need to form beliefs about the persistent component (e.g., by using a Kalman filter) while forecasting $a_{it}$. This implies that innovations to the persistent component are revealed slowly.
Appendix I: Detailed Derivations

I.A Case 1: Both factors chosen under imperfect information

As we show in equation (5) in the text, the firm’s capital choice problem can be written as

$$\max_{K_{it}} \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha_1}} \mathbb{E}_{it} [A_{it}] K_{it}^{\alpha_1} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) R_t K_{it}$$

and optimality requires

$$\alpha \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha_1}} \mathbb{E}_{it} [A_{it}] = R_t$$

$$\Rightarrow \left[ \alpha \left( \frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha_1}} \mathbb{E}_{it} [A_{it}] \right]^{\frac{1}{1-\alpha}} = K_{it}$$

Capital market clearing then implies

$$\int K_{it} di = \left[ \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha_1}} \mathbb{E}_{it} [A_{it}] \right]^{\frac{1}{1-\alpha}} \int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di = K_t$$

$$\Rightarrow \left[ \left( \frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\alpha_1}} \mathbb{E}_{it} [A_{it}] \right]^{\frac{1}{1-\alpha}} \int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di = \frac{K_t}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di}$$

from which we can solve for

$$K_{it} = \frac{\left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} K_t$$

From here, it is straightforward to express firm revenue as

$$P_{it} Y_{it} = K_{it}^{\alpha_1} N^{\alpha_2} Y_t^{\frac{1}{\alpha_1}} A_{it} \left( \frac{\left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left( \mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} \right)^{\alpha}$$
and noting that aggregate revenue must equal aggregate output, we have

\[ Y_t = \int P_t Y_t' \, dt = K_t^{\alpha_1} N^{\alpha_2} Y_t^\frac{1}{\theta} \int A_{it} (E_{it} [A_{it}])^{\frac{1 - \alpha}{2}} \, di \]

or in logs,

\[ y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \log \int A_{it} (E_{it} [A_{it}])^{\frac{1 - \alpha}{2}} \, di - \alpha \log \int (E_{it} [A_{it}])^{\frac{1 - \alpha}{2}} \, di \]

Now, note that under conditional log-normality,

\[ a_{it} | I_{it} \sim N (E_{it} [a_{it}], \sigma^2) \Rightarrow E_{it} [A_{it}] = \exp \left( E_{it} a_{it} + \frac{1}{2} \sigma^2 \right) \]

The true fundamental \( a_{it} \) and its conditional expectation \( E_{it} a_{it} \) are also jointly normal, i.e.

\[
\begin{pmatrix}
a_{it} \\
E_{it} a_{it}
\end{pmatrix}
\sim N
\begin{pmatrix}
\bar{a} \\
\bar{a}
\end{pmatrix},
\begin{pmatrix}
\sigma_a^2 & \sigma_a^2 - \sigma^2 \\
\sigma_a^2 - \sigma^2 & \sigma_a^2 - \sigma^2
\end{pmatrix}
\]

We then have that

\[
\log \int A_{it} (E_{it} [A_{it}])^{\frac{1 - \alpha}{2}} \, di = \log \int \exp \left( a_{it} + \frac{\alpha}{1 - \alpha} E_{it} a_{it} + \frac{1}{2} \frac{\alpha}{1 - \alpha} \sigma^2 \right) \, di
\]

\[
= \frac{1}{1 - \alpha} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \left( \frac{\alpha}{1 - \alpha} \right)^2 (\sigma_a^2 - \sigma^2)
\]

\[
+ \frac{\alpha}{1 - \alpha} (\sigma_a^2 - \sigma^2) + \frac{1}{2} \frac{\alpha}{1 - \alpha} \sigma^2
\]

\[
= \frac{1}{1 - \alpha} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \alpha \left( \frac{2 - \alpha}{(1 - \alpha)^2} \right) (\sigma_a^2 - \sigma^2) + \frac{1}{2} \frac{\alpha}{1 - \alpha} \sigma^2
\]

and

\[
\log \int (E_{it} [A_{it}])^{\frac{1 - \alpha}{2}} \, di = \frac{1}{1 - \alpha} \bar{a} + \frac{1}{2} \left( \frac{1}{1 - \alpha} \right)^2 (\sigma_a^2 - \sigma^2) + \frac{1}{2} \frac{1}{1 - \alpha} \sigma^2
\]

Combining these,

\[
\log \int A_{it} (E_{it} [A_{it}])^{\frac{1 - \alpha}{2}} \, di - \alpha \log \int (E_{it} [A_{it}])^{\frac{1 - \alpha}{2}} \, di = \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{1 - \alpha} - \frac{1}{2} \frac{\alpha}{1 - \alpha} \sigma^2
\]
Substituting and rearranging, we obtain the expressions in (10) and (11) in the text,

\[
y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{1 - \alpha} - \frac{1}{2} \frac{\alpha}{1 - \alpha} V
\]

\[
= \hat{\alpha}_1 k_t + \hat{\alpha}_2 n + \theta \frac{\hat{\alpha}}{\theta - 1} \bar{a} + \frac{1}{2} \left( \theta \frac{\hat{\alpha}}{\theta - 1} \right) \frac{\sigma_a^2}{1 - \alpha} - \frac{1}{2} \theta V
\]

\[
= \hat{\alpha}_1 k_t + \hat{\alpha}_2 n + a
\]

with aggregate productivity is given by \( a = a^* - \frac{1}{2} \theta V \).

It now remains to endogenize \( K_t \). The rental rate in steady state satisfies

\[
R = \frac{1}{\beta} - 1 + \delta.
\]

Then, from the optimality and market clearing conditions, we have

\[
\frac{\alpha_1 N}{\alpha_2 K} = \frac{R}{W} \Rightarrow K \propto W
\]

i.e., the aggregate capital stock is proportional to the wage. To characterize wages, we return to the firm’s profit maximization problem

\[
\max_{K_{it}, N_{it}} Y_t^{\frac{1}{\alpha_1}} E_{it} [A_{it}] K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W N_{it} - RK_{it}
\]

which, after maximizing over capital, can be written as

\[
\max_{N_{it}} (1 - \alpha_1) \left( \frac{\alpha_1}{R} \right)^{\frac{1}{1 - \alpha_1}} \left( Y_t^{\frac{1}{\alpha_1}} E_{it} [A_{it}] N_{it}^{\alpha_2} \right)^{\frac{1}{1 - \alpha_1}} - W N_{it}
\]

Optimality and labor market clearing imply

\[
\left( \frac{\alpha_2}{W} \right)^{\frac{1}{1 - \alpha_1 - \alpha_2}} \left( \frac{\alpha_1}{R} \right)^{\frac{1}{1 - \alpha_1 - \alpha_2}} \left( Y_t^{\frac{1}{\alpha_1}} E_{it} [A_{it}] \right)^{\frac{1}{1 - \alpha_1 - \alpha_2}} = N_{it}
\]

\[
\left( \frac{\alpha_2}{W} \right)^{\frac{1}{1 - \alpha_1 - \alpha_2}} \left( \frac{\alpha_1}{R} \right)^{\frac{1}{1 - \alpha_1 - \alpha_2}} \int \left( Y_t^{\frac{1}{\alpha_1}} E_{it} [A_{it}] \right)^{\frac{1}{1 - \alpha_1 - \alpha_2}} di = N
\]

As before, letting \( \alpha = \alpha_1 + \alpha_2 \), we see that

\[
W \propto \left( \int E_{it} [A_{it}]^{\frac{1}{\alpha_1}} di \right)^{\frac{1}{1 - \alpha_1}} Y_t^{\frac{1}{\alpha_1}}
\]

\[
= \left[ \left( \int \left( \exp \left( E_{it} a_{it} - \frac{1}{2} V \right) \right) di \right)^{\frac{1}{\alpha_1}} Y_t^{\frac{1}{\alpha_1}} \right]^{\frac{1}{1 - \alpha_1}} = \left[ \exp \left( \bar{a} + \frac{1}{2} \left( \sigma_a^2 - \frac{1}{2} V \right) \right) Y_t^{\frac{1}{\alpha_1}} \right]^{\frac{1}{1 - \alpha_1}}
\]

\[
= \exp \left( \bar{a} + \frac{1}{2} \left( \sigma_a^2 - \frac{1}{2} V \right) \right) Y_t^{\frac{1}{\alpha_1}}
\]
or in logs,

\[ w \propto \left( \frac{1}{1 - \alpha_1} \right) \bar{a} + \frac{1}{1 - \alpha_1} \frac{1}{2} \left( \sigma_2^2 - \alpha \bar{V} \right) + \frac{1}{\theta} \frac{1}{1 - \alpha_1} y_t \]

Recalling that

\[ K \propto W \Rightarrow \frac{dW}{dV} = \frac{dW}{d\bar{V}} \]

which, in conjunction with (10) and (11), implies

\[ \frac{dy}{dV} = \hat{\alpha}_1 \left( \frac{dk}{dV} \right) - \frac{1}{2} \theta = \hat{\alpha}_1 \left[ \frac{1}{2} \frac{\alpha}{1 - \alpha} + \frac{1}{\theta} \frac{dy}{dV} \right] - \frac{1}{2} \theta \]

and finally, collecting terms and rearranging, and using the fact that \( \hat{\alpha}_1 = \frac{\theta}{\theta - 1} \alpha \), we obtain

\[ \frac{dy}{dV} = -\frac{1}{2} \theta \frac{1}{1 - \alpha_1} = \frac{dA}{dV} \frac{1}{1 - \alpha_1} \]

**I.B Case 2: Only capital chosen under imperfect information**

The firm’s labor choice problem can be written as

\[ \max_{N_{it}} \frac{1}{W_t} Y_{it}^{\frac{1}{\alpha_1}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} \]

and optimality requires

\[ N_{it} = \left( \frac{\alpha_2}{W_t} Y_{t}^{\frac{1}{\alpha_2}} A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{1 - \alpha_2}} \]

Labor market clearing then implies

\[
\int N_{it} \, di = \int \left( \frac{\alpha_2}{W_t} Y_{t}^{\frac{1}{\alpha_2}} A_{it} K_{it}^{\alpha_1} \right)^{\frac{1}{1 - \alpha_2}} di = N \\
\Rightarrow N_{it} = \left( \frac{A_{it} K_{it}^{\alpha_1}}{\int (A_{it} K_{it}^{\alpha_1})^{\frac{1}{1 - \alpha_2}} di} \right)^{\frac{1}{1 - \alpha_2}} N = \frac{A_{it} K_{it}^{\alpha_1}}{\int A_{it} K_{it}^{\alpha_1} di} N
\]

where we let \( \hat{A}_{it} = A_{it}^{\frac{1}{1 - \alpha_2}} \) and \( \hat{\alpha} = \frac{\alpha_1}{1 - \alpha_2} \). This then implies

\[ W_t = Y_{t}^{\frac{1}{\alpha_2}} A_{it} K_{it}^{\alpha_1 \alpha_2} \left( \frac{\hat{A}_{it} K_{it}^{\alpha_1}}{\int \hat{A}_{it} K_{it}^{\alpha_1} di} N \right)^{\alpha_2 - 1} \]

\[ = \frac{\alpha_2}{N^{1 - \alpha_2}} Y_{t}^{\frac{1}{\alpha_2}} \left( \int \hat{A}_{it} K_{it}^{\alpha_1} di \right)^{1 - \alpha_2} A_{it} K_{it}^{\alpha_1} \left( \frac{1}{\int A_{it} K_{it}^{\alpha_1} di} \right)^{\alpha_2 - 1} \]

\[ = \frac{\alpha_2}{N^{1 - \alpha_2}} Y_{t}^{\frac{1}{\alpha_2}} \left( \int \hat{A}_{it} K_{it}^{\alpha_1} di \right)^{1 - \alpha_2} \]
From here, it is straightforward to express the firm’s capital choice problem as in (8):

$$\max_{K_t} (1 - \alpha_2) \left( \frac{\alpha_2}{W_t} \right)^{\frac{\alpha_2}{1 - \alpha_2}} Y_t \frac{1}{1 - \alpha_2} E_{it} [\hat{A}_{it}] K_{it}^\alpha - R_t K_t$$

Optimality requires

$$R_t = (1 - \alpha_2) \left( \frac{\alpha_2}{W_t} \right)^{\frac{\alpha_2}{1 - \alpha_2}} Y_t \frac{1}{1 - \alpha_2} E_{it} [\hat{A}_{it}] \hat{A}_{it} K_{it}^{\alpha - 1}$$

$$\Rightarrow K_{it} = \left[ \frac{(1 - \alpha_2)\hat{A}_t}{R} \right] \frac{1}{\alpha} \left( \frac{\alpha_2}{W_t} \right)^{\frac{\alpha_2}{1 - \alpha_2}} Y_t \frac{1}{1 - \alpha_2} \frac{1}{1 - \alpha} \left( E_{it} [\hat{A}_{it}] \right)^{\frac{1}{1 - \alpha}}$$

Capital market clearing then implies

$$\int K_{it} di = \left[ \frac{(1 - \alpha_2)\hat{A}_t}{R} \right] \frac{1}{\alpha} \left( \frac{\alpha_2}{W_t} \right)^{\frac{\alpha_2}{1 - \alpha_2}} Y_t \frac{1}{1 - \alpha_2} \frac{1}{1 - \alpha} \int \left( E_{it} [\hat{A}_{it}] \right)^{\frac{1}{1 - \alpha}} di = K_t$$

$$\Rightarrow K_{it} = \frac{\int E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}} di}{\int E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}} di} K_t$$

and we can rewrite the labor choice as

$$N_{it} = \frac{\hat{A}_{it} K_{it}^\alpha}{\int \hat{A}_{it} K_{it}^\alpha di} N = \frac{\hat{A}_{it} \left( E_{it} [\hat{A}_{it}] \right)^{\frac{1}{1 - \alpha}}}{\int \hat{A}_{it} \left( E_{it} [\hat{A}_{it}] \right)^{\frac{1}{1 - \alpha}} di} N$$

Combining the solutions for capital and labor, we can express firm revenue as

$$P_{it} Y_{it} = \left( \frac{E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}}}{\int E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}} di} \right) \left( \frac{\hat{A}_{it} E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}}}{\int \hat{A}_{it} E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}} di} \right) N$$

$$= Y_t \frac{1}{1 - \alpha} K_{it}^{\alpha - 1} N^{\alpha - 2} \left( \frac{E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}}}{\int E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}} di} \right)^{\alpha_1} \left( \frac{\hat{A}_{it} E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}}}{\int \hat{A}_{it} E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}} di} \right)^{\alpha_2}$$

$$= Y_t \frac{1}{1 - \alpha} K_{it}^{\alpha - 1} N^{\alpha - 2} \left( \frac{E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}}}{\int E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}} di} \right)^{\alpha_1} \left( \frac{\hat{A}_{it} E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}}}{\int \hat{A}_{it} E_{it} [\hat{A}_{it}]^{\frac{1}{1 - \alpha}} di} \right)^{\alpha_2}$$

and again using the fact that $y_t = \log \int P_{it} Y_{it} di$, we can write

$$y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n - \alpha_1 \log \int \left( E_{it} [\hat{A}_{it}] \right)^{\frac{1}{1 - \alpha}} di + (1 - \alpha_2) \log \int \hat{A}_{it} \left( E_{it} [\hat{A}_{it}] \right)^{\frac{\alpha}{1 - \alpha}} di$$
Again, we exploit log-normality to obtain

\[
\log \int \hat{A}_t \left( E_{it} \left[ \hat{A}_{it} \right] \right) \frac{1}{\tilde{\sigma}_a^2} \, di = \log \int \exp \left( \tilde{a}_{it} + \frac{\tilde{\alpha}}{1 - \tilde{\alpha}} E_{it} \tilde{a}_{it} + \frac{1}{2} \frac{\tilde{\alpha}^2}{1 - \tilde{\alpha}} \right) \, di \\
= \frac{1}{1 - \tilde{\alpha}} \tilde{a} + \frac{1}{2} \frac{\tilde{\alpha}^2}{1 - \tilde{\alpha}} \left( \sigma^2_{\tilde{a}} - \tilde{\nu} \right) + \frac{1}{2} \frac{\tilde{\alpha}^2}{1 - \tilde{\alpha}} \tilde{\nu}
\]

and similarly,

\[
\log \int \left( E_{it} \left[ \hat{A}_{it} \right] \right) \frac{1}{\tilde{\sigma}_a^2} \, di = \log \int \left( \exp \left( \tilde{a}_{it} + \frac{\tilde{\alpha}}{1 - \tilde{\alpha}} \right) \right) \frac{1}{\tilde{\sigma}_a^2} \, di \\
= \frac{1}{1 - \tilde{\alpha}} \tilde{a} + \frac{1}{2} \frac{\tilde{\alpha}^2}{1 - \tilde{\alpha}} \left( \sigma^2_{\tilde{a}} - \tilde{\nu} \right) + \frac{1}{2} \frac{\tilde{\alpha}^2}{1 - \tilde{\alpha}} \tilde{\nu}
\]

Combining, and using the fact that \( \tilde{a}_{it} = \frac{\alpha_a}{1 - \alpha_2} \):

\[
-\alpha_1 \log \int \left( E_{it} \left[ \hat{A}_{it} \right] \right) \frac{1}{\tilde{\sigma}_a^2} \, di + (1 - \alpha_2) \log \int \hat{A}_t \left( E_{it} \left[ \hat{A}_{it} \right] \right) \frac{1}{\tilde{\sigma}_a^2} \, di \\
= \frac{1 - \alpha_2 - \alpha_1}{1 - \tilde{\alpha}} \tilde{a} + \frac{1}{2} \frac{1}{(1 - \tilde{\alpha})^2} \left( \frac{(1 - \alpha_2)(2\tilde{\alpha} - \tilde{\alpha}_a^2)}{1 - \tilde{\alpha}} \left( \sigma^2_{\tilde{a}} - \tilde{\nu} \right) \right) \\
= \frac{1 - \alpha_2 - \alpha_1}{1 - \tilde{\alpha}} \tilde{a} + \frac{1}{2} \frac{1}{(1 - \tilde{\alpha})^2} \left( \frac{\alpha_1}{1 - \tilde{\alpha}} \sigma^2_{\tilde{a}} - \tilde{\nu} \right) \\
= \tilde{a} + \frac{1}{2} \frac{1}{1 - \tilde{\alpha}} \sigma^2_{\tilde{a}} - \frac{1}{2} \frac{\alpha_1}{1 - \tilde{\alpha}} \tilde{\nu}
\]

Substituting and collecting terms, we obtain expressions (10) and (12) in the text,

\[
y_t = \tilde{a}_1 k_t + \tilde{a}_2 n + \frac{\theta}{\theta - 1} \tilde{\alpha} + \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right)^2 \frac{\sigma^2_{\tilde{a}}}{1 - \tilde{\alpha}} - \frac{1}{2} \left( \theta \tilde{\alpha}_1 + \tilde{\alpha}_2 \right) \tilde{\alpha}_1 \tilde{\nu}
\]

with aggregate productivity given by \( a = a^* - \frac{1}{2} \left( \theta \tilde{\alpha}_1 + \tilde{\alpha}_2 \right) \tilde{\alpha}_1 \tilde{\nu} \).

To endogenize \( K_t \), we begin by characterizing the steady state wage \( W \) in terms of \( \int \hat{A}_t \left( E_{it} \left[ \hat{A}_{it} \right] \right) \frac{1}{\tilde{\sigma}_a^2} \, di \) and \( Y^{1/2} \):

\[
W = \frac{\alpha_2}{N^{1 - \alpha_2}} Y^{1/2} \left( \int \hat{A}_t K^2_{it} \, di \right)^{1 - \alpha_2}
\]

\[
= \frac{\alpha_2}{N^{1 - \alpha_2}} Y^{1/2} \left\{ \int \hat{A}_t \left( \frac{1 - \alpha_2}{R} \frac{\alpha_2}{W} Y^{1/2} \int E_{it} \left[ \hat{A}_{it} \right] \frac{1}{\tilde{\sigma}_a^2} \, di \right)^{1 - \alpha_2} \right\}
\]

\[
= \frac{\alpha_2}{N^{1 - \alpha_2}} Y^{1/2} \left[ \left( \frac{1 - \alpha_2}{R} \right)^{1/2} \frac{\alpha_2}{W} Y^{1/2} \int E_{it} \left[ \hat{A}_{it} \right] \frac{1}{\tilde{\sigma}_a^2} \, di \right]^{1 - \alpha_2}
\]
and rearranging,

\[
W = \left( \frac{\alpha_2}{N^{1-\alpha_2}} \right)^{\frac{1-\hat{\alpha}}{1-\alpha}} \left[ \frac{(1-\alpha_2)\hat{\alpha}}{R} \right]^{\frac{\alpha_1}{1-\alpha}} \alpha_2^{\frac{\alpha_2}{1-\alpha}} \frac{\sigma_a^2}{\bar{\alpha}_1 + 1} \int \hat{A}_t \left( E_{it} \left[ \hat{A}_it \right] \right)^{\frac{\alpha_2}{1-\alpha}} \frac{\sigma_a^2}{\bar{\alpha}_1} Y \right]^{\frac{1}{1-\alpha}} \left\{ \int \hat{A}_it \left( E_{it} \left[ \hat{A}_it \right] \right)^{\frac{\alpha_2}{1-\alpha}} \right\}^{\frac{1}{1-\alpha}} \left\{ \int \hat{A}_it \left( E_{it} \left[ \hat{A}_it \right] \right)^{\frac{\alpha_2}{1-\alpha}} \right\}^{\frac{1}{1-\alpha}} \int \hat{A}_it \left( E_{it} \left[ \hat{A}_it \right] \right)^{\frac{\alpha_2}{1-\alpha}} \right\}
\]

or in logs,

\[
w \propto \left[ \frac{1}{1-\alpha_1} \right] \hat{\alpha} + \frac{1}{2} \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \sigma_a^2 \right) \frac{1}{2} \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \right) \frac{\hat{\alpha}(1-\alpha_2)}{1-\alpha} \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \right)
\]

As before,

\[
K \propto W \quad \Rightarrow \quad \frac{dk}{d\bar{\alpha}} = \frac{dw}{d\bar{\alpha}} = -\frac{1}{2} \left( \frac{\hat{\alpha}(1-\alpha_2)}{1-\alpha} \right) \left( \frac{1}{1-\alpha} \right) + \frac{1}{\bar{\theta} \left( 1-\alpha_1 \right)} \frac{dy}{d\bar{\alpha}}
\]

and substituting into the derivative of aggregate output,

\[
\frac{dy}{d\bar{\alpha}} = \hat{\alpha}_1 \left( \frac{dk}{d\bar{\alpha}} \right) - \frac{1}{2} \left( \bar{\theta} \hat{\alpha}_1 + \hat{\alpha}_2 \right) \hat{\alpha}_1
\]

Finally, collecting terms and rearranging, and using the facts that \(1-\alpha = (1-\hat{\alpha})(1-\alpha_2)\) and \(\left( \frac{\bar{\theta}}{\bar{\theta} + 1-\alpha} \right) \alpha_1 = \hat{\alpha}_1\), we obtain

\[
\frac{dy}{d\bar{\alpha}} = -\frac{1}{2} \left( \bar{\theta} \hat{\alpha}_1 + \hat{\alpha}_2 \right) \hat{\alpha}_1 \left( \frac{1}{1-\alpha_1} \right) = \frac{da}{d\bar{\alpha}} \left( \frac{1}{1-\alpha_1} \right)
\]

### I.C The stock market

Rewriting the marginal investor’s indifference condition in recursive form yields a fixed-point characterization of the price function:

\[
\mathbb{P}(a_{-1}, a, z) = \int \pi (a_{-1}, a, a + \sigma_v z) dH (a|a_{-1}, a + \sigma_v z, a + \sigma_v z)
\]

(30)

Next, we connect expected profits \(\pi(\cdot)\) in the price function to the firm’s problem. For brevity, we only
show the derivation for case 1 (case 2 is very similar). The firm’s profit is

\[
\pi = \left( \frac{N}{K_t} \right)^{\alpha_2} A_{it} \left( \frac{\left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} \right)^{\alpha_1} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) R K_{it}
\]

\[
= \left( \frac{N}{K_t} \right)^{\alpha_2} Y_{it}^{\frac{1}{\alpha}} A_{it} \left( \frac{\left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} \right)^{\alpha_1} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) \left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} R K_{it}
\]

\[
= \left( \frac{N}{K_t} \right)^{\alpha_2} \left( \frac{K_t}{\int \left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} \right)^{\alpha_1} A_{it} \left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} - \left( 1 + \frac{\alpha_2}{\alpha_1} \right) \left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} R K_{it}
\]

\[
= \Gamma_1 A_{it} \left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} - \Gamma_2 \left( E_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}
\]

From the definition of the firm’s information set,

\[
E_{it} [A_{it}] = E [A_{it} | a_{it-1}, a_{it}, e_{it}, a_{it} + \sigma_v z_{it}] = \frac{V}{\sigma^2 \rho} (a_{it-1} - \bar{a})
\]

The profit function \( \pi(\cdot) \) is obtained after integrating out the noise term in the firm’s signal

\[
\pi (a_{it-1}, a_{it}, e_{it}, a_{it} + \sigma_v z_{it}) = \Gamma_1 \int A_{it} \left( E [A_{it} | a_{it-1}, a_{it}, e_{it}, a_{it} + \sigma_v z_{it}] \right)^{\frac{1}{1-\alpha}} d\Phi \left( \frac{e_{it}}{\sigma_e} \right)
\]

\[
- \Gamma_2 \int \left( E [A_{it} | a_{it-1}, a_{it}, e_{it}, a_{it} + \sigma_v z_{it}] \right)^{\frac{1}{1-\alpha}} d\Phi \left( \frac{e_{it}}{\sigma_e} \right)
\]

Given \( \pi(\cdot) \), we solve the functional equation (30) using an iterative procedure with a discrete grid of shock realizations. We then verify that the conjectured threshold and invertibility properties of the equilibrium hold for all points on the grid.

### I.D Other Distortions

Here, we show how a general class of output and capital distortions, \((1 - \tau_{Y_t})\) and \((1 + \tau_{K_t})\), can be mapped into a composite distortion on capital. We start with Case 1, where the firm’s distorted objective is given by, as in Hsieh and Klenow (2009),

\[
\max_{K_{it}, N_{it}} E_{it} \left[ (1 - \tau_{Y_t}) A_{it} K_{it}^{\alpha} N_{it}^{\alpha_2} - W N_{it} - (1 + \tau_{K_t}) R K_{it} \right]
\]

As before, we use (4) to substitute out for \(N_{it}\) and derive the following FOC for \(K_{it}\)

\[
K_{it} = \left\{ \Phi_1 E_{it} \left[ \frac{(1 - \tau_{Y_t}) A_{it}}{(1 + \tau_{K_t})} \right] \right\}^{\frac{1}{\alpha}}
\]
where $\Phi_1$ is a constant. Letting

$$\tau_{it} \equiv \log \frac{1 - \tau_{Y_{it}}}{1 + \tau_{K_{it}}} = \log (1 - \tau_{Y_{it}}) - \log (1 + \tau_{K_{it}})$$

yields equation (19) in the text. Directly, the covariance matrix of $(a_{it}, \tau_{it})$ can be derived using

$$\begin{bmatrix} \sigma_a^2 & \sigma_{a\tau} \\ \sigma_{a\tau} & \sigma_{\tau}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Sigma \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Finally, we can express

$$\tau_{it} = \gamma a_{it} + \varepsilon_{it}$$

where $\gamma \equiv \frac{\sigma_a^2}{\sigma_a^2}$ and $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2 - \gamma^2 \sigma_a^2)$. The marginal revenue product of capital is given by

$$mrp_{K_{it}} = y_{it} - k_{it} = a_{it} + (\alpha - 1) k_{it} = a_{it} - (1 + \gamma) E_t a_{it} - \varepsilon_{it}$$

with a cross-sectional dispersion equal to

$$\sigma_{mrp_k}^2 = \sigma_a^2 + (1 + \gamma)^2 (\sigma_a^2 - \mathbb{V}) - 2 (1 + \gamma) (\sigma_a^2 - \mathbb{V}) + \sigma_\varepsilon^2$$

$$= \mathbb{V} + \gamma^2 (\sigma_a^2 - \mathbb{V}) + \sigma_\varepsilon^2$$

Analogously, for Case 2,

$$K_{it} = \left\{ \Phi_2 E_t \left[ \frac{(1 - \tau_{Y_{it}})^{\frac{1}{\alpha_2}} A_{it}}{(1 + \tau_{K_{it}})} \right] \right\}^{\frac{1}{1 - \alpha}}$$

which yields the following expression for the composite distortion:

$$\tilde{\tau}_{it} \equiv \log \left[ \frac{(1 - \tau_{Y_{it}})^{\frac{1}{\alpha_2}}}{(1 + \tau_{K_{it}})} \right] = \frac{\log (1 - \tau_{Y_{it}})}{1 - \alpha_2} - \log (1 + \tau_{K_{it}})$$

To characterize the stochastic properties of $\tilde{\tau}_{it}$, we follow the same steps as in Case 1.
I.E Identification

**Transitory shocks.** A log-linear approximation of prices (around the deterministic, undistorted case):\(^{84}\)

\[
P \exp(p_{it}) = \tilde{E}_{it}AK^\alpha \exp(a_{it} + \alpha k_{it}) - \tilde{E}_{it}RK \exp(k_{it}) + \beta \tilde{E}_{it}P_{it+1} + \text{Const.}
\]

\[
\Rightarrow P + Pp_{it} \approx AK^\alpha - RK + \beta P + AK^\alpha \tilde{E}_{it}(a_{it} + \alpha k_{it}) - RK\tilde{E}_{it}k_{it} + \beta \tilde{E}_{it}P_{it+1} + \text{Const.}
\]

\[
PP_{it} \approx AK^\alpha \tilde{E}_{it}a_{it} + (\alpha AK^\alpha - RK) \frac{\tilde{E}_{it} \mathbb{E}[a_{it}]}{1-\alpha} + \beta \tilde{E}_{it}P_{it+1} + \text{Const.}
\]

\[
p_{it} = \frac{AK^\alpha}{P} \tilde{E}_{it}a_{it} + \beta \tilde{E}_{it}P_{it+1} + \text{Const.}
\]

where \(\tilde{E}\) denotes the marginal investor’s expectation. We guess

\[
p_{it} = \xi \tilde{E}_{it}a_{it} + \text{Const.}
\]

Substituting,

\[
\xi \tilde{E}_{it}a_{it} + \text{Const.} = \frac{AK^\alpha}{P} \tilde{E}_{it}a_{it} + \beta \xi \tilde{E}_{it}a_{it+1} + \text{Const.} \quad \Rightarrow \quad \xi = \frac{Y}{P} = \frac{1 - \beta}{1 - \alpha}
\]

Now, recall that both of the signals in the marginal investor’s information set are equal to \(u_{it} + \sigma_v z_{it}\). Then,

\[
\tilde{E}_{it}[a_{it}] = \tilde{E}_{it}[\mu_{it}] = \psi (\mu_{it} + \sigma_v z_{it})
\]

where

\[
\psi = \frac{1}{\sigma_u^2} + \frac{1}{\sigma_v^2 + \sigma_z^2}
\]

Combining, and setting \(\kappa = \xi \psi\), yields (22).

---

84. The aggregate constant multiplying revenues is normalized to 1.
Variances (of growth rates):

\[
\sigma_p^2 \equiv \text{Var} (p_{it} - p_{it-1}) = 2\psi^2 \xi^2 \left( \sigma_p^2 + \sigma_e^2 \sigma_z^2 \right) = 2\psi^2 \xi^2 \left( 1 + \frac{\sigma_p^2 \sigma_z^2}{\sigma_p^2} \right) \sigma_u^2
\]

\[
\sigma_k^2 \equiv \text{Var} (k_{it} - k_{it-1}) = \left[ 2 (\phi_1 + \phi_2)^2 \sigma_p^2 + \phi_2^2 \sigma_e^2 \sigma_z^2 + \phi_2^2 \sigma_e \sigma_z \right] \left( \frac{1}{1 - \alpha} \right)^2
\]

\[
= 2 \left( \frac{1}{\sigma_e^2} + \frac{1}{\sigma_z^2} \right)^2 \sigma_a^2 + \left( \frac{1}{\sigma_e^2} \right)^2 \sigma_e^2 + \left( \frac{1}{\sigma_z^2} \right)^2 \sigma_z^2 \sigma_e^2 \left( \frac{1}{1 - \alpha} \right)^2
\]

\[
= 2 \left( \frac{1}{\sigma_e^2} + \frac{1}{\sigma_z^2} \right)^2 \sigma_a^2 = 2 \left( \sigma_p^2 - \sqrt{\sigma_a^2} \right) \left( \frac{1}{1 - \alpha} \right)^2
\]

\[
\sigma_a^2 \equiv \text{Var} (a_{it} - a_{it-1}) = 2\sigma_p^2
\]

Covariances (of growth rates):

\[
\text{Cov} (\Delta p, \Delta k) = 2\xi \psi (\phi_1 + \phi_2) \sigma_p^2 + \phi_2 \sigma_e^2 \sigma_z^2 \left( \frac{1}{1 - \alpha} \right) \]

\[
= 2\xi \psi \left( \frac{1}{\sigma_e^2} + \frac{1}{\sigma_z^2} \right) \sigma_a^2 + \left( \frac{1}{\sigma_e^2} \right)^2 \sigma_e^2 + \left( \frac{1}{\sigma_z^2} \right)^2 \sigma_z^2 \sigma_e^2 \left( \frac{1}{1 - \alpha} \right)
\]

\[
= 2 \left( \frac{1}{\sigma_e^2} + \frac{1}{\sigma_z^2} \right)^2 \sigma_a^2 = 2 \left( \sigma_p^2 - \sqrt{\sigma_a^2} \right) \left( \frac{1}{1 - \alpha} \right)
\]

\[
\text{Cov} (\Delta p, \Delta a) = 2\xi \psi \sigma_a^2
\]

Correlations:

\[
\rho_{pa} = \frac{\text{Cov} (\Delta p, \Delta a)}{\sigma_p \sigma_a} = \frac{2\xi \psi \sigma_a^2}{\sqrt{4\psi^2 \xi^2 \left( \sigma_p^2 + \sigma_e^2 \sigma_z^2 \right) \sigma_a^2}} = \frac{1}{\sqrt{1 + \frac{\sigma_p^2 \sigma_z^2}{\sigma_a^2}}}
\]

\[
\rho_{pk} = \frac{\text{Cov} (\Delta p, \Delta k)}{\sigma_p \sigma_k} = \frac{2\xi \psi \sigma_a^2}{\sqrt{4\psi^2 \xi^2 \left( \sigma_p^2 + \sigma_e^2 \sigma_z^2 \right) \left( \sigma_p^2 - \sqrt{\sigma_a^2} \right)}}
\]

\[
= \frac{1}{\sqrt{\left( 1 + \frac{\sigma_p^2 \sigma_z^2}{\sigma_a^2} \right) \left( 1 - \frac{\sqrt{\sigma_a^2}}{\sigma_p^2} \right)}}
\]

The volatility of returns:

\[
\sigma_p^2 = \xi^2 \left( \frac{\sigma_z^2 + 1}{\sigma_z^2 + 1 + \frac{\sigma_p^2 \sigma_z^2}{\sigma_p^2}} \right)^2 \left( 1 + \frac{\sigma_p^2 \sigma_z^2}{\sigma_p^2} \right) \sigma_u^2 = \xi^2 \left( \frac{\sigma_z^2 + 1}{\sigma_z^2 + 1 + \frac{\sigma_p^2 \sigma_z^2}{\sigma_p^2}} \right)^2 \frac{1}{\rho_{pa}} \sigma_p^2
\]
Permanent shocks. We start with the price function, (suppressing the time subscript and normalizing the GE term multiplying revenues to 1),

\[ P(a_{-1}, a, z) = \tilde{E}(e^a K^\alpha - RK) + \beta \tilde{E}P(a, a', z') \]

where, as before, \( \tilde{E} \) denotes the marginal investor’s expectation. For small information frictions, profits are approximately

\[ e^a K^\alpha - RK \approx e^{a - \alpha} \]

Substituting,

\[ P(a_{-1}, a, z) \approx \tilde{E}e^{a - \alpha} + \beta \tilde{E}P(a, a', z') \]

Guess:

\[ P(a_{-1}, a, z) = \hat{P} \tilde{E}e^{a - \alpha} \]

To verify the guess

\[ \hat{P}\tilde{E}e^{a - \alpha} \approx \tilde{E}e^{a - \alpha} + \beta \hat{P}\tilde{E}(u', z') \cdot \tilde{E}e^{a - \alpha} \]

\[ = \left( 1 + \beta \hat{P}\tilde{E}\Phi(u', z') \right) \tilde{E}e^{a - \alpha} \]

where \( \tilde{E}' \) denotes the expectation of marginal investor in the following period, which in turn is a function \( \Phi \) of the future shock realizations \( (u', z') \). Since \((u', z')\) are iid, our guess is verified. Then,

\[ \log P(a_{-1}, a, z) \approx \text{Const.} + \frac{1}{1 - \alpha} \tilde{E}a \]

or equivalently,

\[ p_{it-1} \approx \frac{1}{1 - \alpha} \tilde{E}_{it-1}a_{it-1} + \text{const.} \]

Next, when \( \rho = 1 \), firm and investor beliefs are given by

\[ E_{it}a_{it} = a_{it-1} + E_{it} \mu_{it} \quad \tilde{E}_{it}a_{it} = a_{it-1} + \tilde{E}_{it} \mu_{it}, \]
which leads to

\[ p_{it} = \frac{1}{1-\alpha} a_{it-1} + \frac{1}{1-\alpha} \psi (u_{it} + \sigma_v z_{it}) + \text{Const} \]

\[ (1-\alpha) k_{it} = a_{it-1} + \phi_1 (\mu_{it} + e_{it}) + \phi_2 (\mu_{it} + \sigma_v z_{it}) + \text{Const}. \]

where the coefficients \( \psi, \phi_1 \) and \( \phi_2 \) are the same as in the i.i.d. case. In growth rates,

\[ \Delta a_{it} = \mu_{it} \]

\[ \Delta p_{it} = \frac{1}{1-\alpha} (a_{it-1} - a_{it-2}) + \frac{1}{1-\alpha} \psi (\mu_{it} + \sigma_v z_{it} - \mu_{it-1} - \sigma_v z_{it-1}) \]

\[ = \frac{1}{1-\alpha} (1-\psi) \mu_{it-1} + \frac{1}{1-\alpha} \psi \mu_{it} + \xi \psi \sigma_v (z_{it} - z_{it-1}) \]

\[ (1-\alpha) \Delta k_{it} = (a_{it-1} - a_{it-2}) + (\phi_1 + \phi_2) (\mu_{it} - \mu_{it-1}) + \phi_1 (e_{it} - e_{it-1}) + \phi_2 \sigma_v (z_{it} - z_{it-1}) \]

\[ = (1 - \phi_1 - \phi_2) \mu_{it-1} + (\phi_1 + \phi_2) \mu_{it} + \phi_1 (e_{it} - e_{it-1}) + \phi_2 \sigma_v (z_{it} - z_{it-1}) \]

Second moments (of growth rates):

\[ \sigma^2_p = \left( \frac{1}{1-\alpha} \right)^2 (1-\psi)^2 \sigma^2_\mu + \left( \frac{1}{1-\alpha} \right)^2 \psi^2 \sigma^2_v^2 + 2 \left( \frac{1}{1-\alpha} \right)^2 \psi^2 \sigma^2_v \sigma^2_z \]

\[ = \left( \frac{1}{1-\alpha} \right)^2 \sigma^2_\mu \left[ 1 - 2\psi + 2\psi^2 (1 + \frac{\sigma^2_v}{\sigma^2_\mu}) \right] \]

\[ (31) \]

\[ \sigma^2_k = \left( \frac{1}{1-\alpha} \right)^2 \sigma^2_\mu (1 - \phi_1 - \phi_2)^2 \sigma^2_\mu + (\phi_1 + \phi_2) \sigma^2_\mu + 2\phi_1^2 \sigma^2_v + 2\phi_2^2 \sigma^2_v \sigma^2_z \]

\[ = \left( \frac{1}{1-\alpha} \right)^2 \left[ \sigma^2_\mu - 2 (\sigma^2_\mu - \nu) + 2\nu^2 \sigma^2_\mu - 4\nu + 2\frac{\nu^2}{\sigma^2_\nu} + 2\frac{\nu^2}{\sigma^2_v} + 2\frac{\nu^2}{\sigma^2_v \sigma^2_z} \right] \]

\[ (32) \]

\[ (33) \]

Thus, the variance of investment is invariant to uncertainty! This is because, with persistence, undoing the effects of past forecast errors also contributes to investment volatility. When shocks are permanent, this exactly offsets the contemporaneous dampening that occurs due to imperfect information. Thus, with permanent shocks, uncertainty shifts the timing of the response of capital to fundamentals, but not the
overall magnitude. Now,

\[
\text{Cov}(\Delta p, \Delta a) = \frac{1}{1 - \alpha} \psi \sigma_{\mu}^2
\]

\[
\text{Cov}(\Delta p, \Delta k) = \left( \frac{1}{1 - \alpha} \right)^2 \left( (1 - \psi)(1 - \phi_1 - \phi_2) \sigma_{\mu}^2 + \psi (\phi_1 + \phi_2) \sigma_{\mu}^2 + 2 \psi \phi_2 \sigma_{\mu}^2 \sigma_z^2 \right)
\]

\[
= \left( \frac{1}{1 - \alpha} \right)^2 \left( (1 - \psi - \phi_1 - \phi_2) \sigma_{\mu}^2 + 2 \psi (\phi_1 + \phi_2) \sigma_{\mu}^2 + 2 \psi \phi_2 \sigma_{\mu}^2 \sigma_z^2 \right)
\]

\[
= \left( \frac{1}{1 - \alpha} \right)^2 \left( (1 - \psi - \phi_1 - \phi_2) \sigma_{\mu}^2 + 2 \psi (\sigma_{\mu}^2 - \psi) + 2 \psi \right)
\]

\[
= \left( \frac{1}{1 - \alpha} \right)^2 (\psi + \psi \sigma_{\mu}^2)
\]

Directly,

\[
\text{Cov}(\Delta p, \Delta k) - \left( \frac{1}{1 - \alpha} \right) \text{Cov}(\Delta p, \Delta a) = \left( \frac{1}{1 - \alpha} \right)^2 V
\]

\[
\rho_{pk} \sigma_p \sigma_k - \left( \frac{1}{1 - \alpha} \right) \rho_{pa} \sigma_p \sigma_a =
\]

\[
\rho_{pk} \sigma_p \sigma_u \left( \frac{1}{1 - \alpha} \right) - \left( \frac{1}{1 - \alpha} \right) \rho_{pa} \sigma_p \sigma_u =
\]

\[
(\rho_{pk} - \rho_{pa}) \left( \frac{1}{1 - \alpha} \right) \sigma_p \sigma_{\mu} =
\]

Re-arranging yields the expression in the text.

\[
\frac{V}{\sigma_{\mu}^2} = \frac{(\rho_{pk} - \rho_{pa}) \sigma_p}{\sigma_{\mu}} (1 - \alpha)
\]

Correlations:

\[
\rho_{pk} = \frac{\psi + \frac{V}{\sigma_{\mu}}}{\sqrt{1 - 2 \psi + 2 \psi^2 \left(1 + \frac{\sigma_{\mu}^2 \sigma_z^2}{\sigma_{\mu}^2} \right)}}
\]

\[
\rho_{pa} = \frac{\psi}{\sqrt{1 - 2 \psi + 2 \psi^2 \left(1 + \frac{\sigma_{\mu}^2 \sigma_z^2}{\sigma_{\mu}^2} \right)}} = \frac{1}{\sqrt{(1 - \psi)^2 + 1 + 2 \frac{\sigma_{\mu}^2 \sigma_z^2}{\sigma_{\mu}^2}}}
\]

Then,

\[
\frac{\rho_{pk}}{\rho_{pa}} = 1 + \frac{1}{\psi} \frac{V}{\sigma_{\mu}^2} \Rightarrow \psi = \frac{\frac{V}{\sigma_{\mu}^2}}{\frac{\rho_{pk}}{\rho_{pa}} - 1} = \frac{\rho_{pa}}{\eta}
\]

58
Given $\psi$, we then use the expression for $\rho_{pa}$ to solve for $\frac{\sigma_z^2 \sigma^2_v}{\mu}$:

$$\rho_{pa}^2 = \frac{1}{\left(\frac{\eta}{\rho_{pa}} - 1\right)^2 + 1 + 2\frac{\sigma_z^2 \sigma^2_v}{\mu^2}}$$

$$\Rightarrow \frac{\sigma_z^2 \sigma^2_v}{\mu^2} = \frac{1}{2\rho_{pa}^2} - \frac{1}{2} \left(\frac{\eta}{\rho_{pa}} - 1\right)^2 - \frac{1}{2} = \frac{(1 - \eta^2)}{2\rho_{pa}^2} + \eta - 1$$

Along with the definition of $\psi$, this allows us to disentangle $\sigma_z^2$ and $\sigma_v^2$.

### I.F Other distortions

Note that distortions have only a second order effect on profits. Therefore, our expressions for stock prices, which use first-order approximations, are unaffected both under iid and permanent shocks. In other words, the only moments affected by distortions are $\text{Cov}(p, k)$ and $\sigma_k$. With only correlated distortions, both these moments are scaled by $1 + \gamma$. Uncorrelated distortions have no effect on $\text{Cov}(p, k)$ but increase $\sigma_k$.

**Transitory shocks.** With correlated distortions, since both $\text{Cov}(p, k)$ and $\sigma_k$ are scaled up by the same factor, $\rho_{pk}$ remains unchanged:

$$\rho_{pk} = \frac{\text{Cov}(p, k)}{\sigma_p \sigma_k} = \frac{2\xi \psi \sigma_k^2 (1 + \gamma)}{\sqrt{4 \psi^2 \xi^2 (\sigma_\mu^2 + \sigma_v^2 \sigma_z^2) (\sigma_u^2 - \eta^2)}}$$

$$= \frac{2\xi \psi \sigma_k^2}{\sqrt{4 \psi^2 \xi^2 (\sigma_\mu^2 + \sigma_v^2 \sigma_z^2) (\sigma_u^2 - \eta^2)}} \cdot \frac{1}{\sqrt{1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2} \left(1 - \frac{\psi}{\sigma_k^2}\right)}}$$

the same as before.

With uncorrelated distortions,

$$\rho_{pk} = \frac{\text{Cov}(p, k)}{\sigma_p \sigma_k} = \frac{2\xi \psi \sigma_k^2}{\sqrt{4 \psi^2 \xi^2 (\sigma_\mu^2 + \sigma_v^2 \sigma_z^2) (\sigma_u^2 + \sigma_k^2 - \eta^2)}}$$

$$= \frac{1}{\sqrt{\left(1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}\right) \left(1 - \frac{\psi}{\sigma_k^2} + \frac{\sigma_k^2}{\sigma_\mu^2}\right)}} \cdot \frac{\rho_{pa}}{\sqrt{1 - \frac{\psi}{\sigma_k^2} + \frac{\sigma_k^2}{\sigma_\mu^2}}}$$

so that using (26) will yield an underestimate of $\frac{\psi}{\sigma_k^2}$ by an amount $\frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}$. 

59
Permanent shocks. With correlated distortions, equation (35) becomes

\[
\frac{Cov(\Delta k, \Delta p)}{1 + \gamma} = \left( \frac{1}{1 - \alpha} \right) Cov(\Delta p, \Delta a) + \left( \frac{1}{1 - \alpha} \right)^2 \mathbb{V}
\]

\[
\frac{\rho_{pk}\sigma_p\sigma_k}{1 + \gamma} = \frac{1}{1 - \alpha} \rho_{pa}\sigma_p\sigma_\mu + \left( \frac{1}{1 - \alpha} \right)^2 \mathbb{V}
\]

\[
\left( \frac{1}{1 - \alpha} \right) \rho_{pk}\sigma_p\sigma_\mu = \mathbb{V}
\]

yielding the same relationship between \( \mathbb{V} \) and the other moments as before. In other words, using equation (29) leads us to the correct \( \mathbb{V} \). If distortions are uncorrelated, we still have

\[
\rho_{pk}\sigma_p\sigma_k = \left( \frac{1}{1 - \alpha} \right) \rho_{pa}\sigma_p\sigma_\mu + \left( \frac{1}{1 - \alpha} \right)^2 \mathbb{V}
\]

but now \( \sigma_k = \frac{1}{1 - \alpha} \sqrt{\sigma_\mu^2 + 2\sigma_\varepsilon^2} \). Substituting,

\[
\rho_{pk}\sigma_p\sigma_\mu \sqrt{1 + 2\left( \frac{\sigma_\varepsilon^2}{\sigma_\mu^2} \right)} = \rho_{pa}\sigma_p\sigma_\mu + \frac{1}{1 - \alpha} \mathbb{V}
\]

so that now, the formula \((\rho_{kp}\sigma_p - \rho_{pa}\sigma_p)\sigma_\mu\) underestimates \( \mathbb{V} \).

**I.G Correlated signals**

Here, we present alternative, more direct derivations of equations (26) and (29), which rely only on conditional normality and not on the correlation structure.

Transitory shocks. We start with the i.i.d. case. Then, \( p_{it} \) is an i.i.d. random variable, uncorrelated with past and future realizations of \( a_{it} \).

\[
Cov(\Delta p_{it}, \Delta a_{it}) = Cov(p_{it}, a_{it}) + Cov(p_{it-1}, a_{it-1}) = 2Cov(p_{it}, a_{it})
\]

\[
= 2Cov(p_{it}, E_{it}a_{it} + \varepsilon_{it}) = 2Cov(p_{it}, E_{it}a_{it}) + 2Cov(p_{it}, \varepsilon_{it})
\]

where \( \varepsilon_{it} \) denotes the firm’s forecast error, which is uncorrelated with firm information - in particular, with any element in the firm information set. Therefore, \( Cov(p_{it}, \varepsilon_{it}) = 0 \). Note that this is true independent of the correlation structure between \( p_{it} \) and the other elements of that information set. Now, we use
\[ \text{Cov}(p_{it}, k_{it}) = \frac{1}{1-\alpha} \text{Cov}(p_{it}, \mathbb{E}_{it} a_{it}) \] and divide both sides of the equation by \( \sigma_p \sigma_a \) to get

\[
\frac{\text{Cov}(\Delta p_{it}, \Delta a_{it})}{\sigma_p \sigma_a} = \frac{2 \text{Cov}(p_{it}, k_{it})(1-\alpha)\sigma_k}{\sigma_p \sigma_a} = \frac{\text{Cov}(\Delta p_{it}, \Delta k_{it})(1-\alpha)\sigma_k}{\sigma_p \sigma_k \sigma_a} \]

\[
\Rightarrow \frac{\rho_{pa}}{\rho_{pk}} = \frac{(1-\alpha)\sigma_k}{\sigma_a}
\]

Since

\[
\sigma_k^2 = \left(\frac{1}{1-\alpha}\right)^2 \text{E}_{it}^2(a_{it}) = \left(\frac{1}{1-\alpha}\right)^2 \left(\sigma_{p_{it}}^2 - \mathbb{V}\right)
\]

\[
= \left(\frac{1}{1-\alpha}\right)^2 \sigma_{p_{it}}^2 \left(1 - \frac{\mathbb{V}}{\sigma_{p_{it}}^2}\right) = \left(\frac{1}{1-\alpha}\right)^2 \sigma_{p_{it}}^2 (a_{it} - a_{it-1}) (1 - \frac{\mathbb{V}}{\sigma_{p_{it}}^2})
\]

\[
\Rightarrow \frac{\sigma_k}{\sigma_a} = \frac{1}{1-\alpha} \sqrt{1 - \frac{\mathbb{V}}{\sigma_{p_{it}}^2}}
\]

Combining yields (26).

**Permanent shocks.** Next, consider the case with \( \rho = 1 \). Investment now becomes

\[
(1-\alpha) \Delta k_{it} = a_{it-1} + \mathbb{E}_{it} u_{it} - a_{it-2} + \mathbb{E}_{it-1} u_{it-1}
\]

\[
= \mathbb{E}_{it} u_{it} + u_{it-1} - \mathbb{E}_{it-1} u_{it-1}
\]

\[
p_{it} = \frac{1}{1-\alpha} \left(a_{it-1} + \mathbb{E}_{it} u_{it}\right) = \frac{1}{1-\alpha} \left(a_{it-1} + \tilde{\psi} \mathbb{E}_{it}^p (u_{it})\right)
\]

where \( \mathbb{E}_{it}^p (u_{it}) \) is the expectation conditional on \( p_{it} \) alone. Here, we make use of the fact that the marginal investor’s expectation is proportional to \( \mathbb{E}_{it}^p \). The constant of proportionality \( \tilde{\psi} \) depends on the variance parameters, but, as we will see, will play no role in the determination of \( \mathbb{V} \). Stock returns are given by:

\[
(1-\alpha) \Delta p_{it} = a_{it-1} + \tilde{\psi} \mathbb{E}_{it}^p u_{it} - a_{it-2} - \tilde{\psi} \mathbb{E}_{it-1}^p u_{it-1}
\]

\[
= u_{it-1} + \tilde{\psi} \mathbb{E}_{it}^p u_{it} - \tilde{\psi} \mathbb{E}_{it-1}^p u_{it-1}
\]

\[
= \tilde{\psi} \mathbb{E}_{it}^p u_{it} + \left(1 - \tilde{\psi}\right) u_{it-1} + \tilde{\psi} (u_{it-1} - \mathbb{E}_{it-1}^p u_{it-1})
\]

Define

\[
\omega_{it-1} = \mathbb{E}_{it-1} u_{it-1} - \mathbb{E}_{it-1}^p u_{it-1}
\]

Since \( \omega_{it-1} \) is in the firm’s information set, it must be orthogonal to the firm’s forecast error \( \varpi_{it-1} = u_{it-1} - \mathbb{E}_{it-1} u_{it-1} \). We can use this to write the forecast error component in stock returns as the sum of two
orthogonal components

\[
u_{it-1} - E_{it-1}^p u_{it-1} = (u_{it-1} - E_{it} u_{it-1}) + \omega_{it-1} = \varpi_{it-1} + \omega_{it-1}\]

Substituting into the expression for the growth rates, we get

\[
(1 - \alpha) (k_{it} - k_{it-1}) = E_{it}^p u_{it} + \omega_{it} + \varpi_{it-1}
\]

\[
(1 - \alpha) (p_{it} - p_{it-1}) = \tilde{\psi} E_{it}^p u_{it} + (1 - \tilde{\psi}) u_{it-1} + \tilde{\psi} (\varpi_{it-1} + \omega_{it-1})
\]

Covariances

\[
Cov(\Delta p_{it}, \Delta a_{it}) = \left( \frac{1}{1 - \alpha} \right) \tilde{\psi} Var(E_{it}^p u_{it})
\]

\[
Cov(\Delta p_{it}, \Delta k_{it}) = \left( \frac{1}{1 - \alpha} \right)^2 \tilde{\psi} Var(E_{it}^p u_{it}) + \left( \frac{1}{1 - \alpha} \right) (1 - \tilde{\psi}) Cov(u_{it-1}, \varpi_{it-1})
\]

\[
+ \left( \frac{1}{1 - \alpha} \right)^2 \tilde{\psi} Var(\varpi_{it-1})
\]

\[
= \left( \frac{1}{1 - \alpha} \right) Cov(\Delta p, \Delta a) + \left( \frac{1}{1 - \alpha} \right)^2 \left( 1 - \tilde{\psi} \right) V + \left( \frac{1}{1 - \alpha} \right)^2 \tilde{\psi} V
\]

\[
= \left( \frac{1}{1 - \alpha} \right) Cov(\Delta p, \Delta a) + \left( \frac{1}{1 - \alpha} \right)^2 V
\]

Which gives us

\[
\rho_{kp} \sigma_p \sigma_k = \left( \frac{1}{1 - \alpha} \right) \rho_{pa} \sigma_p \sigma_a + \left( \frac{1}{1 - \alpha} \right)^2 V
\]

\[
\frac{1}{1 - \alpha} \rho_{kp} \sigma_p \sigma_a = \left( \frac{1}{1 - \alpha} \right) \rho_{pa} \sigma_p \sigma_a + \left( \frac{1}{1 - \alpha} \right)^2 V
\]

\[
\rho_{kp} \sigma_p \sigma_a = \rho_{pa} \sigma_p \sigma_a + \frac{1}{1 - \alpha} V
\]

Re-arranging yields equation (29).

**APPENDIX II: DATA**

We use annual data on firm-level production variables and stock market returns from Compustat North America (for the US) and Compustat Global (for China and India). For each country, we exclude duplicate observations (firms with multiple observations within a single year), firms not incorporated within that
country, and firms not reporting in local currency. We build three year production periods as the average of firm sales and capital stock over non-overlapping 3-year horizons (i.e., \( K_{2012} = \frac{K_{2010} + K_{2011} + K_{2012}}{3} \), and analogously for sales). We measure the capital stock using gross property, plant and equipment (PPEGT in Compustat terminology), defined as the “valuation of tangible fixed assets used in the production of revenue.” We then calculate investment as the change in the firm’s capital stock relative to the preceding period. We construct the firm fundamental \( a_{it} \) as the log of value-added less the relevant \( \alpha \) (which depends on the case) multiplied by the log of the capital stock. We compute value-added from revenues using a share of intermediates of 0.5 (our data does not generally include measures of intermediate inputs). Finally, we first-difference the investment and fundamental series to compute investment growth and changes in fundamentals.

Stock returns are constructed as the change in the firm’s stock price over the three year period, adjusted for splits and dividend distributions. We follow the procedure outlined in the Compustat manual. In Compustat terminology, and as in the text, using \( p_{it} \) as shorthand for log returns, returns for the US are computed as

\[
p_{it} = \log \left( \frac{PRCCM_{it} \times TRFM_{it}}{AJEXM_{it}} \right) - \log \left( \frac{PRCCM_{it-1} \times TRFM_{it-1}}{AJEXM_{it-1}} \right)
\]

where periods denote 3 year spans (i.e., returns for 2012 are calculated as the adjusted change in price between 2009 and 2012), PRCCM is the firm’s stock price, and TRFM and AJEXM adjustment factors needed to translate prices to returns from the Compustat monthly securities file. Data are for the last trading day of the firm’s fiscal year so that the timing lines up with the production variables just described. The calculation is analogous for China and India, with the small caveat that global securities data come daily, so that the Compustat variables are PRCCD, TRFD, and AJEXDI, where “D” denotes days. Again, the data are for the last trading day of the firm’s fiscal year. Table A.1 displays summary statistics for 2012: the number of firms in each country, the share of GDP they account for, and average sales and capital stock in US dollars. GDP data is from the World Bank Development Indicators database. Foreign currencies are converted to US dollars using the exchange rate on December 31, 2012.

To extract the firm-specific variation in our variables, we regress each on a time fixed-effect and work with the residual. This eliminates the component of each series common to all firms in a time period and leaves only the idiosyncratic variation. As described in the text, we limit our sample to a single cross-section, namely 2012, and finally, we trim the 2% tails of each series. It is then straightforward to compute the target moments, i.e., \( \sigma_p^2 \), \( \rho_{pi} \), and \( \rho_{pa} \). As described in the text, we lag returns by one period, so that, e.g., \( \rho_{pi} \) is the correlation of 2006-09 returns with investment growth from 2009-12.

To estimate the parameters governing the evolution of firm fundamentals, i.e., the persistence \( \rho \) and
Table A.1

Sample Statistics

<table>
<thead>
<tr>
<th></th>
<th>No. of firms</th>
<th>Share of GDP</th>
<th>Avg. Sales ($M)</th>
<th>Avg. PPEGT ($M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>6618</td>
<td>0.42</td>
<td>2792</td>
<td>2117</td>
</tr>
<tr>
<td>China</td>
<td>2270</td>
<td>0.12</td>
<td>851</td>
<td>639</td>
</tr>
<tr>
<td>India</td>
<td>2028</td>
<td>0.21</td>
<td>388</td>
<td>323</td>
</tr>
</tbody>
</table>

variance of the innovations $\sigma^2_{\mu}$, we perform the autoregression implied by (1). Here we use annual observations on $a_{it}$ at a 3-year frequency in order to simplify issues of time aggregation. We estimate the process using our data from 2012 and 2009. We include a time fixed-effect in order to isolate the idiosyncratic component of the innovations in $a_{it}$. Differences in firm fiscal years means that different firms within the same calendar year are reporting data over different time periods, and so the time fixed-effect incorporates both the reporting year and month. The results from this regression deliver an estimate for $\rho$ and $\sigma^2_{\mu}$, from which we trim the 1% tails.

Our leverage adjustment is as follows: we assume that claims to firm profits are sold to investors in the form of both debt and equity in a constant proportion (within each country). This implies that the payoff from an equity claim is $S_{it} = \mathcal{V}_{it} - D_{it-1}$, where $\mathcal{V}_{it}$ is the value of the unlevered firm and $D_{it-1} = d \mathbb{E}_t [\mathcal{V}_{it}]$, where $d \in (0,1)$ represents the share of expected firm value in the hands of debt-holders. In other words, firm value is allocated to investors as a debt claim that pays off a constant fraction of its ex-ante expected value and as a residual claim to equity holders. The change in value of an equity claim is then equal to $\Delta S_{it} = \Delta \mathcal{V}_{it}$ and dividing both sides by the ex-ante expected value of the claim (i.e., the price) $\bar{\mathcal{S}} = \bar{\mathcal{V}} - d \bar{\mathcal{V}}$, where $\bar{\mathcal{V}} = \mathbb{E}_t [\mathcal{V}_{it}]$, gives returns as $\frac{\Delta S_{it}}{\bar{S}} = \frac{\Delta \mathcal{V}_{it}}{(1-d)\bar{\mathcal{V}}}$. Taking logs and computing variances shows $\sigma^2_{\mathcal{V}_{it}} = (1-d)^2 \sigma^2_{S_{it}}$, i.e., the volatility in (unlevered) firm value is a fraction $(1-d)^2$ of the volatility in (levered) equity returns. To assign values to $d$ in each country, we examine the debt-asset and debt-equity ratios of the set of firms in Compustat over the period 2006-2009. Because these vary to some degree from year to year and depend to some extent on the precise approach taken (i.e., whether we use debt-assets or debt-equity and whether we compute average ratios or totals), we simply take the approximate midpoints of the ranges for each country, which are about 0.30 for the US and India and 0.16 for China, leading to adjustment factors of about 0.5 and 0.7, respectively.
APPENDIX III: QUANTITATIVE EXERCISES

III.A Case 1 vs Case 2

Table A.2 reports the moments and parameters using labor data for US firms (measured as total employment), rather than capital. Moments are constructed as in the baseline analysis, using employment rather than capital. We denote the correlation of stock returns with changes in employment growth $\rho_{pn}$.

Table A.2

<table>
<thead>
<tr>
<th>Target moments</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{pn}$</td>
<td>$\rho$</td>
</tr>
<tr>
<td>$0.17$</td>
<td>$0.87$</td>
</tr>
</tbody>
</table>

III.B Adjustment Costs

When firms are fully informed but subject to capital adjustment costs, the dynamic optimization problem of the firm is characterized by the value function:

$$V\left(\tilde{A}_{it}, K_{it-1}\right) = \max_{K_{it}, N_{it}} G\tilde{A}_{it}K_{it}^{\tilde{\alpha}} - K_{it} - \zeta K_{it-1} \left(\frac{K_{it}}{K_{it-1}} - 1\right)^2 + \beta\mathbb{E}V\left(\tilde{A}_{it+1}, (1 - \delta) K_{it}\right)$$

To characterize $G$ and $\tilde{A}_{it}$, we start with the firm’s labor choice problem

$$\max_{N_{it}} P_{it}Y_{it} - WN_{it} = \max_{N_{it}} Y_{it}^{\frac{1}{\alpha_2}} A_{it}K_{it}^{\alpha_1} N_{it}^{\alpha_2} - WN_{it}$$

where $\alpha$’s are as defined in (2). Optimizing over $N_{it}$ and substituting back into the objective gives

$$P_{it}Y_{it} - WN_{it} = (1 - \alpha_2) \left(\frac{\alpha_2}{W}\right)^{\frac{1}{\alpha_2}} Y_{it}^{\frac{1}{\alpha_2}} \tilde{A}_{it}K_{it}^{\tilde{\alpha}}$$

where $\tilde{\alpha}$ and $\tilde{A}_{it}$ are defined as in case 2 in the text. We can then solve for

$$G = (1 - \alpha_2) \left[\left(\frac{\alpha_2}{W}\right)^{\alpha_2} Y_{it}^{\frac{1}{\alpha_2}}\right]^{\frac{1}{1 - \alpha_2}}$$
Next, using the firm’s labor optimality condition, labor market clearing implies

\[
\left( \frac{\alpha_2}{W} \right)^{1-\alpha_2} Y_t^{\frac{1}{\theta}} \int \hat{A}_{it} K_{it}^{\alpha_2} di = N
\]

from which we can solve for

\[
\left( \frac{\alpha_2}{W} \right)^{\alpha_2} Y_t^{\frac{1}{\theta}} = \left[ \int \hat{A}_{it} K_{it}^{\alpha_2} di \right]^{\frac{1-\alpha_2}{\theta-1}} N^\frac{\theta}{\theta-1} \alpha_2
\]

and so

\[
G = (1 - \alpha_2) \left[ \int \hat{A}_{it} K_{it}^{\alpha_2} di \right]^{\frac{1-\alpha_2}{\theta-1}} N^\frac{\theta}{\theta-1} \alpha_2
\]

Note that under full information, the production side of the economy is completely decoupled from stock markets, so we can now solve the model laid out above. Starting with a guess for the general equilibrium term \(G\), we solve for the value functions (using a standard iterative procedure and a discretized grid for capital), simulate to obtain the steady state distributions and verify that our initial guess for \(G\) is consistent with that distribution. If not, we update the guess and iterate until convergence.

We then solve for stock prices, again using the conjecture that the informed investors follow a threshold rule. Proceeding exactly as in the baseline model, we can then derive the following functional equation for the price:

\[
P(a_{it-1}, k_{it-1}, a_{it}, z_{it}) = \int \pi(a_{it}, k_{it-1}) H(a_{it}|a_{it-1}, a_{it} + \sigma_v z_{it}, P_{it})
+ \beta \int \tilde{P}(a_{it}, k^*(a_{it}, k_{it-1})) H(a_{it}|a_{it-1}, a_{it} + \sigma_v z_{it}, P_{it})
\]

The model is parameterized as described in the text. Table A.3 reports the full set of target moments and parameter estimates. Notice that the first two columns are identical to those in Table II (case 2). \(\sigma_v^2\) denotes the variance of investment rates, which is used to pin down the size of the adjustment cost \(\zeta\).

**Table A.3**

**TARGETS AND PARAMETERS - ADJUSTMENT COST MODEL**

<table>
<thead>
<tr>
<th>Target moments</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_{pa})</td>
<td>(\sigma_v)</td>
</tr>
<tr>
<td>(\sigma_p^2)</td>
<td>(\sigma_k^2)</td>
</tr>
<tr>
<td>US</td>
<td>0.18</td>
</tr>
<tr>
<td>China</td>
<td>0.06</td>
</tr>
<tr>
<td>India</td>
<td>0.08</td>
</tr>
</tbody>
</table>
III.C Measurement Error

With measurement error in revenues, the following equations relate the observed variables and moments (denoted with a hat) with their true counterparts (no hat). In this case, the affected moments are $\rho$, $\sigma^2$, and $\rho_{pa}$.

\[ \hat{\rho} = \frac{\text{Cov}(a_{it} + \epsilon_{it}^y, a_{it-1} + \epsilon_{it-1}^y)}{\sigma_a^2 + \sigma_{cy}^2} = \frac{\text{Cov}(a_{it}, a_{it-1})}{\sigma_a^2 + \sigma_{cy}^2} = \rho \left( \frac{\hat{\sigma}_a^2 - \sigma_{cy}^2}{\sigma_a^2} \right) \]

\[ \hat{\mu}_{it} = \hat{a}_{it} - \rho \hat{a}_{it-1} - \rho \hat{\epsilon}_{it-1} = a_{it} + \epsilon_{it}^y - \rho a_{it-1} - \rho \epsilon_{it-1}^y + (\rho - \hat{\rho}) a_{it-1} + (\rho - \hat{\rho}) \epsilon_{it-1}^y \]

\[ \Rightarrow \hat{\sigma}_\mu^2 = \sigma_\mu^2 + \sigma_{cy}^2 + (\rho - \hat{\rho})^2 \sigma_a^2 + \rho^2 \sigma_{cy}^2 \]

\[ = \sigma_\mu^2 + \sigma_{cy}^2 + \left( \frac{\sigma_{cy}^2}{\sigma_a^2} \right)^2 \rho^2 \sigma_a^2 + \rho^2 \sigma_{cy}^2 \]

\[ = \sigma_\mu^2 + \sigma_{cy}^2 + \rho^2 \sigma_{cy}^2 \left( 1 + \frac{\sigma_{cy}^2}{\sigma_a^2 - \sigma_{cy}^2} \right) \]

\[ \hat{\rho}_{pa} = \frac{\text{Cov}(\Delta p_{it}, \hat{a}_{it} - \hat{a}_{it-1})}{\sigma_p \hat{\sigma}_{\Delta a}} = \frac{\text{Cov}(\Delta p_{it}, a_{it} - a_{it-1}) \sigma_{\Delta a}}{\sigma_p \hat{\sigma}_{\Delta a}} \]

\[ = \rho_{pa} \frac{\sigma_{\Delta a}^2 - 2 \sigma_{cy}^2}{\sigma_{\Delta a}^2} \]

The corrections to the observed moments when capital is measured with error are similar and omitted for brevity. Table A.4 reports the moments and parameters after adjusting for measurement error for the cases in Table X.

III.D Sectoral Analysis

Our sectoral analysis in Section V.A makes use of data from I/B/E/S Guidance and Compustat. We obtain from I/B/E/S data on firm forecasts of annual revenues and the corresponding realizations. We examine forecasts that are made at approximately a 1 year horizon, specifically, between 10 and 14 months prior to the reporting date. There are very few forecasts available at longer horizons. For forecasts that include a range, we take the midpoint. In order to associate I/B/E/S firms with industry codes and additionally to use a consistent sample of firms from the two sources, we merge the I/B/E/S data with Compustat. We are then able to place each I/B/E/S firm into an SIC code. We examine only firms who are present both in I/B/E/S and Compustat. In this way, we are able to use exactly the same set of firms to construct the sectoral indicator of uncertainty from I/B/E/S and to estimate $V$ from Compustat. We include the period
### Table A.4

**Adjusted Moments and Parameters - Measurement Error**

<table>
<thead>
<tr>
<th>Case 2 - 10% error in y</th>
<th>Target moments</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_{pi}$</td>
<td>$\rho_{pa}$</td>
</tr>
<tr>
<td>US</td>
<td>0.23</td>
<td>0.22</td>
</tr>
<tr>
<td>China</td>
<td>0.16</td>
<td>0.07</td>
</tr>
<tr>
<td>India</td>
<td>0.25</td>
<td>0.09</td>
</tr>
<tr>
<td>Case 2 - 25% error in y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.23</td>
<td>0.36</td>
</tr>
<tr>
<td>China</td>
<td>0.16</td>
<td>0.10</td>
</tr>
<tr>
<td>India</td>
<td>0.25</td>
<td>0.14</td>
</tr>
<tr>
<td>Case 2 - 10% error in k</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.35</td>
<td>0.20</td>
</tr>
<tr>
<td>China</td>
<td>0.22</td>
<td>0.06</td>
</tr>
<tr>
<td>India</td>
<td>0.32</td>
<td>0.08</td>
</tr>
<tr>
<td>Case 1 - 10% error in y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.24</td>
<td>0.11</td>
</tr>
<tr>
<td>China</td>
<td>0.16</td>
<td>0.02</td>
</tr>
<tr>
<td>India</td>
<td>0.26</td>
<td>0.00</td>
</tr>
<tr>
<td>Case 1 - 25% error in y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.24</td>
<td>0.18</td>
</tr>
<tr>
<td>China</td>
<td>0.16</td>
<td>0.03</td>
</tr>
<tr>
<td>India</td>
<td>0.26</td>
<td>0.01</td>
</tr>
<tr>
<td>Case 1 - 10% error in k</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.37</td>
<td>0.11</td>
</tr>
<tr>
<td>China</td>
<td>0.22</td>
<td>0.02</td>
</tr>
<tr>
<td>India</td>
<td>0.33</td>
<td>0.00</td>
</tr>
</tbody>
</table>


Once the sample is constructed, we group firms into 2-digit SIC industry groups and work on a sector-by-sector basis. Sectoral classifications are listed in Table A.5, along with the associated SIC codes and the number of firm-year observations from each of our data sources. The set of firms is the same across the samples; the number of firm-year observations differs since there may not be forecasts for each firm-year observation in Compustat, and vice versa, there may not be sufficient data from Compustat to include a firm-year observation despite having a forecast from I/B/E/S. There are two sector groups where we do not have sufficient data from I/B/E/S - Agriculture, Forestry, Fishing, and Mining. Within each sector, for each forecast-realization pair from I/B/E/S, we compute the log forecast error as $\ln(\text{forecast}) - \ln(\text{actual})$. To extract the idiosyncratic component of the forecast error, we regress the errors on a time fixed-effect and work with the residual. We trim the 1% tails of the errors and compute the variance of the forecast errors, i.e., the mean squared forecast error. This is our indicator of uncertainty at the sectoral level.
moments are computed following the same procedure as for our baseline analysis. The moments are now constructed on a sector-by-sector basis and over the period 1998-2013 where we eliminate time-specific factors (e.g., inflation) by extracting time fixed-effects from all series. Table A.5 reports the mean squared error of firm forecasts from I/B/E/S, the empirical moments from Compustat, the resulting parameter estimates, and our estimate of $V$, all on a sectoral basis.

**III.E Time-Series Analysis**

We construct the empirical moments in our time-series analysis following the same procedure as for our baseline. Moments are computed year-by-year from 1998 to 2012 on a balanced panel of firms which have the necessary data for all years. Because calculation of the moments in each year require 9 years worth of data, firms must have data back to 1990 to be included. There are about 785 firms in our final sample. Table A.6 reports the empirical moments and estimated parameter values for each year.
### Table A.5

**Sectoral Uncertainty - Moments and Parameters**

<table>
<thead>
<tr>
<th>SIC Range</th>
<th>No. of Observations</th>
<th>Target Moments</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \rho_{pi} )</td>
<td>( \rho_{pu} )</td>
</tr>
<tr>
<td>Retail Trade</td>
<td>52-59</td>
<td>0.004</td>
<td>0.35</td>
</tr>
<tr>
<td>FIRE</td>
<td>60-69</td>
<td>0.007</td>
<td>0.17</td>
</tr>
<tr>
<td>Services</td>
<td>70-89</td>
<td>0.011</td>
<td>0.28</td>
</tr>
<tr>
<td>Construction</td>
<td>15-19</td>
<td>0.014</td>
<td>0.36</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>20-39</td>
<td>0.014</td>
<td>0.30</td>
</tr>
<tr>
<td>Trans and PU</td>
<td>40-49</td>
<td>0.015</td>
<td>0.37</td>
</tr>
<tr>
<td>Wholesale Trade</td>
<td>50-51</td>
<td>0.015</td>
<td>0.10</td>
</tr>
</tbody>
</table>
### Table A.6

**Time-Series of Uncertainty - Moments and Parameters**

<table>
<thead>
<tr>
<th>Target moments</th>
<th>Parameters</th>
<th>ρ</th>
<th>σ_p^2</th>
<th>σ_ρ</th>
<th>σ_μ</th>
<th>σ_π</th>
<th>σ_ν</th>
<th>σ_ζ</th>
<th>V</th>
<th>V Detrended</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998 0.28 0.20 0.29</td>
<td>0.85 0.34 0.32 0.35 2.22 0.050 -0.013</td>
<td>0.19</td>
<td>0.15 0.27</td>
<td>0.91</td>
<td>0.35 0.33 0.79 0.94 0.052 -0.010</td>
<td>0.052 -0.010</td>
<td>0.057 -0.003</td>
<td>0.071 0.012</td>
<td>0.073 0.016</td>
<td>0.073 0.018</td>
</tr>
<tr>
<td>2000 0.27 0.12 0.27</td>
<td>0.93 0.33 0.40 1.12 0.63 0.057 -0.003</td>
<td>0.32 0.09 0.33</td>
<td>0.91</td>
<td>0.33 0.50 0.62 1.83 0.071 0.012</td>
<td>0.073 0.016</td>
<td>0.073 0.018</td>
<td>0.073 0.016</td>
<td>0.073 0.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002 0.28 0.13 0.38</td>
<td>0.88 0.34 0.47 0.36 3.40 0.073 0.016</td>
<td>0.28 0.12 0.47</td>
<td>0.95</td>
<td>0.34 0.49 0.45 2.98 0.073 0.018</td>
<td>0.073 0.018</td>
<td>0.073 0.018</td>
<td>0.073 0.018</td>
<td>0.073 0.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003 0.28 0.12 0.47</td>
<td>0.95 0.32 0.34 0.71 1.29 0.050 -0.003</td>
<td>0.27 0.17 0.42</td>
<td>0.95</td>
<td>0.29 0.30 0.39 2.02 0.040 -0.012</td>
<td>0.040 -0.012</td>
<td>0.040 -0.012</td>
<td>0.040 -0.012</td>
<td>0.040 -0.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2004 0.23 0.10 0.41</td>
<td>0.93 0.28 0.35 0.47 2.45 0.046 -0.004</td>
<td>0.30 0.14 0.27</td>
<td>0.97</td>
<td>0.31 0.40 1.54 0.40 0.052 0.004</td>
<td>0.052 0.004</td>
<td>0.052 0.004</td>
<td>0.052 0.004</td>
<td>0.052 0.004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007 0.32 0.10 0.26</td>
<td>0.95 0.29 0.41 1.38 0.46 0.048 0.000</td>
<td>0.30 0.14 0.27</td>
<td>0.97</td>
<td>0.32 0.40 1.62 0.43 0.056 0.011</td>
<td>0.056 0.011</td>
<td>0.056 0.011</td>
<td>0.056 0.011</td>
<td>0.056 0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008 0.30 0.14 0.27</td>
<td>0.97 0.32 0.40 1.62 0.43 0.056 0.011</td>
<td>0.26 0.18 0.25</td>
<td>0.95</td>
<td>0.29 0.33 1.45 0.34 0.040 -0.004</td>
<td>0.040 -0.004</td>
<td>0.040 -0.004</td>
<td>0.040 -0.004</td>
<td>0.040 -0.004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011 0.20 0.12 0.36</td>
<td>0.90 0.27 0.28 0.36 2.77 0.037 -0.005</td>
<td>0.21 0.15 0.32</td>
<td>0.97</td>
<td>0.27 0.28 0.74 0.86 0.035 -0.006</td>
<td>0.035 -0.006</td>
<td>0.035 -0.006</td>
<td>0.035 -0.006</td>
<td>0.035 -0.006</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### REFERENCES


### Table I

**Parameterization - Summary**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.90</td>
</tr>
<tr>
<td>$\hat{\alpha}_1$</td>
<td>Capital share</td>
<td>0.33</td>
</tr>
<tr>
<td>$\hat{\alpha}_2$</td>
<td>Labor share</td>
<td>0.67</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution</td>
<td>6</td>
</tr>
<tr>
<td><strong>Country-specific</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence of fundamentals</td>
<td>Estimates of (1): $a_{it} = (1 - \rho) \bar{a} + \rho a_{it-1} + \mu_{it}$</td>
</tr>
<tr>
<td>$\sigma_{\mu}$</td>
<td>Shocks to fundamentals</td>
<td>$\rho_{pi}$</td>
</tr>
<tr>
<td>$\sigma^2_v$</td>
<td>Firm private signal</td>
<td>$\rho_{pa}$</td>
</tr>
<tr>
<td>$\sigma^2_v$</td>
<td>Investor private signal</td>
<td>$\sigma^2_p$</td>
</tr>
<tr>
<td>$\sigma^2_z$</td>
<td>Noise trading</td>
<td>$\sigma^2_p$</td>
</tr>
</tbody>
</table>
### Table II

**Target Moments and Parameter Estimates**

<table>
<thead>
<tr>
<th></th>
<th>Target moments</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_{pi}$ $\rho_{pa}$ $\sigma^2_p$</td>
<td>$\rho$ $\sigma_{\mu}$ $\sigma_c$ $\sigma_v$ $\sigma_z$</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.23 0.18 0.23</td>
<td>0.92 0.45 0.39 0.37 3.50</td>
</tr>
<tr>
<td>China</td>
<td>0.16 0.06 0.14</td>
<td>0.78 0.51 0.67 0.74 4.24</td>
</tr>
<tr>
<td>India</td>
<td>0.25 0.08 0.23</td>
<td>0.93 0.53 1.04 0.69 4.36</td>
</tr>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.24 0.10 0.23</td>
<td>0.88 0.46 0.63 0.65 3.16</td>
</tr>
<tr>
<td>China</td>
<td>0.15 0.02 0.14</td>
<td>0.75 0.53 0.74 1.18 3.14</td>
</tr>
<tr>
<td>India</td>
<td>0.26 0.00 0.22</td>
<td>0.88 0.55 1.39 1.69 4.14</td>
</tr>
<tr>
<td></td>
<td>$\hat{\sigma}$</td>
<td>$\hat{\sigma}^2$</td>
</tr>
<tr>
<td>----------------</td>
<td>----------------</td>
<td>-------------------</td>
</tr>
<tr>
<td><strong>Case 2 ($\alpha = 0.62$)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.08</td>
<td>0.41</td>
</tr>
<tr>
<td>China</td>
<td>0.16</td>
<td>0.63</td>
</tr>
<tr>
<td>India</td>
<td>0.22</td>
<td>0.77</td>
</tr>
<tr>
<td><strong>Case 1 ($\alpha = 0.83$)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.13</td>
<td>0.63</td>
</tr>
<tr>
<td>China</td>
<td>0.18</td>
<td>0.65</td>
</tr>
<tr>
<td>India</td>
<td>0.26</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>$\theta = 4$</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>--------------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>$V/\sigma_p^2$</td>
<td>$\frac{V}{\sigma_{\text{error}}^2}$</td>
</tr>
<tr>
<td>US</td>
<td>0.05</td>
<td>0.25</td>
</tr>
<tr>
<td>China</td>
<td>0.13</td>
<td>0.52</td>
</tr>
<tr>
<td>India</td>
<td>0.17</td>
<td>0.60</td>
</tr>
</tbody>
</table>
### Table V

**The Importance and Sources of Learning**

<table>
<thead>
<tr>
<th></th>
<th>$\Delta V$</th>
<th>$\frac{\Delta V}{\sigma_V}$</th>
<th>$\Delta a$</th>
<th>$\Delta y$</th>
<th>Share from source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Private</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>$-0.12$</td>
<td>$-0.59$</td>
<td>$0.05$</td>
<td>$0.08$</td>
<td>$0.92$</td>
</tr>
<tr>
<td>China</td>
<td>$-0.10$</td>
<td>$-0.37$</td>
<td>$0.04$</td>
<td>$0.06$</td>
<td>$0.96$</td>
</tr>
<tr>
<td>India</td>
<td>$-0.06$</td>
<td>$-0.23$</td>
<td>$0.03$</td>
<td>$0.04$</td>
<td>$0.89$</td>
</tr>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>$-0.08$</td>
<td>$-0.37$</td>
<td>$0.23$</td>
<td>$0.35$</td>
<td>$0.91$</td>
</tr>
<tr>
<td>China</td>
<td>$-0.10$</td>
<td>$-0.35$</td>
<td>$0.30$</td>
<td>$0.45$</td>
<td>$0.96$</td>
</tr>
<tr>
<td>India</td>
<td>$-0.04$</td>
<td>$-0.14$</td>
<td>$0.12$</td>
<td>$0.19$</td>
<td>$0.96$</td>
</tr>
</tbody>
</table>

*Notes.* The left hand panel shows the total effect of learning, i.e., from all sources: the reduction in $V$ and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The right hand panel shows the relative shares of private and market sources in total learning.
Table VI

The Contributions of Market and Private Information

<table>
<thead>
<tr>
<th>Case</th>
<th>Market Information</th>
<th>Private Information</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Both sources</td>
<td>Only market</td>
</tr>
<tr>
<td></td>
<td>$\Delta V$</td>
<td>$\Delta a$</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>$-0.021$</td>
<td>$0.002$</td>
</tr>
<tr>
<td>China</td>
<td>$-0.010$</td>
<td>$0.001$</td>
</tr>
<tr>
<td>India</td>
<td>$-0.019$</td>
<td>$0.002$</td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>$-0.021$</td>
<td>$0.013$</td>
</tr>
<tr>
<td>China</td>
<td>$-0.009$</td>
<td>$0.008$</td>
</tr>
<tr>
<td>India</td>
<td>$-0.005$</td>
<td>$0.004$</td>
</tr>
</tbody>
</table>

Notes: The table shows the reductions in $V$ due to market and private information and the corresponding gains in aggregate TFP (hence the pattern of negatives and positives). Columns ‘Both sources’ show the effects of each source (market/private) of learning in the presence of the other. Columns ‘Only market’ and ‘Only private’ show the effect of each when it is the only source of information.
### Table VII

**The Effects of a US Information Structure**

<table>
<thead>
<tr>
<th></th>
<th>Case 2</th>
<th>Case 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta V$</td>
<td>$\Delta V$</td>
</tr>
<tr>
<td><strong>Market Information</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>$-0.01$</td>
<td>$-0.05$</td>
</tr>
<tr>
<td>India</td>
<td>$-0.02$</td>
<td>$-0.07$</td>
</tr>
<tr>
<td><strong>Private Information</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>$-0.07$</td>
<td>$-0.26$</td>
</tr>
<tr>
<td>India</td>
<td>$-0.12$</td>
<td>$-0.43$</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>$-0.02$</td>
<td>$-0.09$</td>
</tr>
<tr>
<td>India</td>
<td>$-0.05$</td>
<td>$-0.19$</td>
</tr>
</tbody>
</table>

*Notes.* The table shows the reduction in $V$ and the corresponding gains in the aggregates (hence the pattern of negatives and positives) in India and China from having access to US-quality financial market information (top panel), private information (middle panel) and facing US fundamental shocks (bottom panel).
### Table VIII

**Uncertainty with Correlated Information**

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_e$</th>
<th>$\sigma_v$</th>
<th>$\sigma_z$</th>
<th>$\mathcal{V}$</th>
<th>Baseline $\mathcal{V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.37</td>
<td>0.17</td>
<td>8.01</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>China</td>
<td>0.60</td>
<td>0.63</td>
<td>4.65</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>India</td>
<td>0.79</td>
<td>0.45</td>
<td>6.37</td>
<td>0.19</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Notes. The table shows parameter estimates and the resulting extent of uncertainty when investors are assumed to observe a noisy signal of the firm’s private signal. All parameters not displayed retain the same values as the baseline case.
### Table IX

**A Full-Information Adjustment Cost Model**

<table>
<thead>
<tr>
<th></th>
<th>Relative Correlation</th>
<th>Implied Uncertainty</th>
<th>Baseline Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>$\forall$</td>
</tr>
<tr>
<td>US</td>
<td>0.05</td>
<td>-0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>China</td>
<td>0.10</td>
<td>-0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>India</td>
<td>0.17</td>
<td>-0.02</td>
<td>0.08</td>
</tr>
</tbody>
</table>

*Notes.* The table shows the relative correlation of returns with investment growth and returns with fundamentals ($\rho_{pi} - \rho_{pa}$), both as computed from the data and as predicted by the full-information adjustment cost model, as well as the implied extent of uncertainty from the latter and from the baseline model. $\rho_{pa}$ is a targeted moment and so is the same in the data and model.
### Table X

**The Effect of Measurement Error**

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Error in $y$ - 10%</th>
<th>Error in $y$ - 25%</th>
<th>Error in $k$ - 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>$\frac{\sigma_v}{\sigma_{\mu}}$</td>
<td>$\frac{\sigma_v}{\sigma_{\mu}}$</td>
<td>$\frac{\sigma_v}{\sigma_{\mu}}$</td>
<td>$\frac{\sigma_v}{\sigma_{\mu}}$</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.08</td>
<td>0.05</td>
<td>0.01</td>
<td>0.11</td>
</tr>
<tr>
<td>China</td>
<td>0.16</td>
<td>0.14</td>
<td>0.10</td>
<td>0.17</td>
</tr>
<tr>
<td>India</td>
<td>0.22</td>
<td>0.17</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.13</td>
<td>0.12</td>
<td>0.07</td>
<td>0.17</td>
</tr>
<tr>
<td>China</td>
<td>0.18</td>
<td>0.18</td>
<td>0.17</td>
<td>0.21</td>
</tr>
<tr>
<td>India</td>
<td>0.26</td>
<td>0.25</td>
<td>0.22</td>
<td>0.27</td>
</tr>
</tbody>
</table>

*Notes.* The table shows the extent of uncertainty under various assumption on measurement error. The first panel corresponds to the baseline estimates; the second panel assumes 10% error in $y_{it}$; the third panel assumes 25% error in $y_{it}$; the last panel assumes 10% error in $k_{it}$.
Figure I

Identification - Quantitative Model
Figure II

Uncertainty at the Sectoral Level
Notes. The top panel displays annual estimates of $V$. The bottom panel shows annual estimates of $V$ and annual estimates of micro uncertainty from Jurado, Ludvigson, and Ng (2013). All series are detrended and normalized by their standard deviation.

Figure III

Uncertainty Through Time