The Tail that Wags the Economy: Belief-driven Business Cycles and Persistent Stagnation

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Main idea

No one knows the distribution of aggregate shocks

- Re-estimate beliefs as new data arrives
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Belief revisions are extremely persistent

- As are their effects on aggregate economic activity
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This paper:

Why do some recessions have large, long-lived effects?
Unusual events $\rightarrow$ large belief changes $\rightarrow$ persistent effects
Application: Recent US Experience

Trend computed using 1952-2007

\[
\ln(\text{GDP per capita}, \ 1952 = 1)
\]

\[
0.5, 1, 1.5, 2, 2.5\}

12\%
Application: Recent US Experience

The Great Recession $\rightarrow$ Increase in tail risk $\rightarrow$ Lower $K, Y, N$ etc.

- Quantitatively successful in explaining the persistent decline in output
- Consistent with financial market data and popular narratives
Application: Recent US Experience

The Great Recession → Increase in tail risk → Lower $K$, $Y$, $N$ etc.

- Quantitatively successful in explaining the persistent decline in output
- Consistent with financial market data and popular narratives

A tail risk index from financial markets (SKEW)

Constructed from out-of-the-money put options on S&P 500
Key elements

• Neo-classical production economy with one modification
  • → Re-estimating distribution of aggregate shocks

• Beliefs as a source of internal propagation
  • → Permanent effects from iid shocks

• Beliefs estimated using observed macro data
  • → Empirical discipline, compatible with quantitative equilibrium models.

• Non-parametric approach to estimation
  • → Flexible, avoid distributional assumptions, tail risk vs uncertainty,
Outline

- Belief formation
- Economic environment
- Numerical results
Estimating beliefs

- Consider an iid shock, $\phi_t$, with unknown distribution $g$

- Information set: finite history of shock realizations $\{\phi_{t-s}\}_{s=0}^{n_t-1}$

- **Goal:** a flexible specification that can capture tail risk

- We use a non-parametric estimator: the Gaussian kernel density

$$
\hat{g}_t(x) = \frac{1}{n_t \kappa} \sum_{s=0}^{n_t-1} \Omega \left( \frac{x - \phi_{t-s}}{\kappa} \right)
$$
Example: Returns to Non-residential Capital (1950-2007)

\[ r_t = \frac{(d_t + p_t)}{p_{t-1}} \]

(a) Capital Returns

(b) Estimated beliefs

(c) Future beliefs

Future shocks from \( \hat{g}_{2009} \) → Beliefs are **martingales**, i.e. \( \mathbb{E}_t[\hat{g}_{t+j}|\mathcal{I}_t] \approx \hat{g}_t \)

**Effects are permanent**
Example: Returns to Non-residential Capital (1950-2007)

\[ r_t = \frac{d_t + p_t}{p_{t-1}} \]

(a) Capital Returns

(b) Estimated beliefs

(c) Future beliefs

Future shocks from \( \hat{g}_{2007} \) → Transitory belief revisions, i.e. \( \mathbb{E}_t[\hat{g}_{t+j}|\mathcal{I}_t] \neq \hat{g}_t \)

Effects are still quite persistent
Model

From Gourio (2012, 2013)

- **Production:** \( y_{it} = k_{it}^{\alpha} l_{it}^{1-\alpha} \)

- **Aggregate shocks to capital 'quality':** \( k_{it} = \phi_t \hat{k}_{it} \)

- **Law of motion** \( \hat{k}_{it+1} = k_{it}(1 - \delta) + x_{it} \)

\( \phi_t \sim iid \ g(\cdot) \)
Model

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- **Credit and labor Markets**
  - Firms borrow with 1-period defaultable debt (Eaton-Gersovitz, 1981)
  - Labor is hired in advance, i.e. before observing shocks \( \rightarrow \) operating leverage
  - Idiosyncratic shocks (iid) \( \rightarrow \) positive default in equilibrium

- **Preferences:** Representative HH with EZ preferences over \( C_t = \frac{L_t^{1+\gamma}}{1+\gamma} \)
**Model**

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- **Preferences:** Representative HH with EZ preferences over  \( C_t - \frac{L_t^{1+\gamma}}{1+\gamma} \)

Distribution  \( g \) unknown to all agents

- **Beliefs**
  - At each date, observe  \( \{\phi_1, \ldots, \phi_t\} \)
  - Apply Gaussian kernel density estimator →  \( \hat{g}_t \)
Measurement and Calibration

Why capital quality shocks?

- Most direct way to generate volatile $R^k_t \equiv \frac{\phi_t^{\alpha} \hat{k}_t^{\alpha} l_t^{1-\alpha} - w_t l_t}{\hat{k}_t} + (1 - \delta) \phi_t$

- Simple measurement strategy: **Capital gains $\rightarrow \phi_t \rightarrow$ Beliefs**

Calibration

- Preferences: Risk aversion = 10, IES = 2, Frisch = 2
- Credit: Leverage = 0.5, default rate = 0.02
Measurement and Calibration

Why capital quality shocks?

- Most direct way to generate volatile $R^k_t \equiv \frac{\phi_t^\alpha \hat{k}_t^{\alpha} (1 - \alpha)}{\hat{k}_t} - w_t l_t + (1 - \delta) \phi_t$
- Simple measurement strategy: **Capital gains $\rightarrow \phi_t \rightarrow Beliefs$**

Data: Non-financial assets (Flow of Funds)

- Commercial real estate ($\sim 55\%$), equipment and software
- Market value ($P^k_t K_t$) and historical cost ($\rightarrow P^k_{t-1} \hat{K}_t$)

$$\phi_t = \frac{K_t}{\hat{K}_t} = \left( \frac{P^k_t K_t}{P^k_{t-1} \hat{K}_t} \right) \left( \frac{PINDEX^k_{t-1}}{PINDEX^k_t} \right)$$

Calibration

- Preferences: Risk aversion = 10, IES = 2, Frisch = 2
- Credit: Leverage = 0.5, default rate = 0.02
Estimating beliefs: Aggregate capital quality shocks

- I.i.d. shocks to effective capital: \( K_t = \phi_t \hat{K}_t \)

Last panel: Mean and 2-std dev bands for \( \hat{g}_{2039} \) (drawing from \( \hat{g}_{2009} \))

Large negative shock \( \rightarrow \) Persistent increase in tail risk
Effect of the Great Recession

Strategy

1. Start at ‘steady state’ of $\hat{g}_{2007}$ (estimated using 1950-2007 data)

2. Feed in the actual shocks from 2008-09 and estimate $\hat{g}_{2009}$

\[
(\phi_{2008}, \phi_{2009}) = (0.93, 0.84)
\]

3. Simulate future paths drawing from $\hat{g}_{2009}$ (Baseline) and $\hat{g}_{2007}$ (Extension)

4. Compute updated beliefs, aggregate $K$, $Y$, $N$ along each path

5. Plot the mean future path of aggregates
Results: Mean future path of aggregates (Baseline)

A 12% drop in GDP (on average)

Each panel plots the mean path across 5000 simulations
Results: Model (Baseline) vs Data

Model captures declines in output, labor

Data: Deviations from log-linear, pre-crisis trend
What if beliefs were not re-estimated?

No belief revisions $\rightarrow$ investment boom and recovery

No learning: Agents know the distribution $\hat{g}_{2009}$ from the beginning (rational expectations)
Implications for asset prices

<table>
<thead>
<tr>
<th></th>
<th>Change</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td><strong>Returns and spreads</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit Spreads</td>
<td>0.02%</td>
<td>0.14%</td>
<td></td>
</tr>
<tr>
<td>Risk free rate</td>
<td>-0.54%</td>
<td>-1.42%</td>
<td></td>
</tr>
<tr>
<td>Equity Premium</td>
<td>1.41%</td>
<td>3.27%</td>
<td></td>
</tr>
<tr>
<td>Equity (Market value)/Assets</td>
<td>1%</td>
<td>21%</td>
<td></td>
</tr>
<tr>
<td><strong>Tail risk on equity</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E \left[ (R^e - \bar{R}^e)^3 \right] $</td>
<td>-0.27%</td>
<td>-0.28%</td>
<td></td>
</tr>
<tr>
<td>$\text{Prob}(R^e - \bar{R}^e \leq -0.3)$</td>
<td>1.48%</td>
<td>2.23%</td>
<td></td>
</tr>
</tbody>
</table>

What if future shocks are drawn from $\hat{g}_{2007}$?

Shocks drawn from $\hat{g}_{2009}$ (solid blue) versus $\hat{g}_{2007}$ (dashed green).

Belief revisions extremely persistent, though not permanent
Why are some recessions more persistent?

Small shocks → Negligible long-run effects

Small shock: Realizations equal to 2001-’02
What if the learning sample includes pre-1950 data?

Two issues

- Data on \( \phi_t \) ? Discounting of old data ?
What if the learning sample includes pre-1950 data?

Two issues

- Data on $\phi_t$? Discounting of old data?

Strategy: Use the 1950-2009 sample as a proxy for 1890-1949

- Great Depression: \[ \{\phi_{1929}, \phi_{1930}\} = \{\phi_{2008}, \phi_{2009}\}^\varepsilon, \quad \varepsilon \in \{1, 2\} \]

- Weights: Observation in $t - s$ is given a weight $\lambda^s$, $\lambda \leq 1$
What if the learning sample includes pre-1950 data?

Two issues

- Data on $\phi_t$? Discounting of old data?

Strategy: Use the 1950-2009 sample as a proxy for 1890-1949

- Great Depression: $\{\phi_{1929}, \phi_{1930}\} = \{\phi_{2008}, \phi_{2009}\}^\epsilon$, $\epsilon \in \{1, 2\}$
- Weights: Observation in $t - s$ is given a weight $\lambda^s$, $\lambda \leq 1$

More data (+ modest discounting) yields similar results
What if we assumed a parametric form?

- Density
- GDP

2007 vs. 2009

Kernel, DGP 2007
Normal, DGP 2007

2010 2020 2030
-0.2 -0.15 -0.1 -0.05 0 0.05

GDP

2010 2020 2030
-0.2 -0.15 -0.1 -0.05 0 0.05
Role of mean, risk, debt

Constant mean: Imposes $\bar{\phi}_t = \bar{\phi}_{2007}$
No risk aversion: Sets $\eta = \psi = 0$
No debt: An identical economy with $\chi = 1$, i.e. all equity financing

Changes in the mean → about half of the long-run effects

Risk aversion and debt amplify effects of belief revisions
Conclusion

• Obviously, no one knows the true distribution of shocks

• New data permanently reshapes our assessment of macro risks

• → A new perspective on the current prolonged stagnation

• Tools for embedding and disciplining beliefs in quantitative macro models
Debt and Large vs Small Shocks

- Debt makes economy more sensitive to tail probabilities
- Tail probabilities are most sensitive to new information

Debt amplifies belief revisions from large shocks
The economic mechanism

<table>
<thead>
<tr>
<th></th>
<th>% Change in GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark model</td>
<td>−0.12</td>
</tr>
<tr>
<td><strong>Counterfactuals</strong></td>
<td></td>
</tr>
<tr>
<td>Constant mean</td>
<td>−0.06</td>
</tr>
<tr>
<td>No risk aversion</td>
<td>−0.07</td>
</tr>
<tr>
<td>No debt</td>
<td>−0.09</td>
</tr>
</tbody>
</table>

Mean, risk and debt all important for long-run effects
What could temper the permanence of our effects?

If long-run beliefs differ significantly from $\hat{g}_{2009}$, e.g. because of

- New, positive data, e.g. a long period without crises
- The distribution $g$ changes over time, i.e. regime shifts
Contribution to the Literature

Secular stagnation:

  - We add: *new mechanism, acting through belief revisions*

Belief-driven business cycles

- Belief shocks: Gourio (2012), Angeletos and La'O (2013), Bloom (2009)...
  - We add: *endogenous belief revisions, persistence*

- Learning models: Johannes et. al. (2012), Cogley and Sargent (2005)...
  - We add: *production, non-parametric learning*

- Endogenous uncertainty: Fajgelbaum et.al. (2014), Straub and Ulbricht (2013)...
  - We add: *empirical discipline on beliefs, larger effects*
Calibration

Strategy: Match aggregate and cross-sectional moments of the US economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.91</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\eta$</td>
<td>10</td>
<td>Risk aversion</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.50</td>
<td>1/Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.50</td>
<td>1/Frisch elasticity</td>
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<tr>
<td>$\zeta$</td>
<td>1</td>
<td>Labor disutility</td>
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<tr>
<td>Technology:</td>
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<tr>
<td>$\alpha$</td>
<td>0.40</td>
<td>Capital share</td>
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<td>$\delta$</td>
<td>0.03</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.25</td>
<td>Idiosyncratic volatility</td>
</tr>
<tr>
<td>Debt:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>1.06</td>
<td>Tax advantage of debt</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.70</td>
<td>Recovery rate</td>
</tr>
</tbody>
</table>
Why are the long run effects so large? An approximate calculation

Consider the deterministic steady state without debt

\[
\frac{1}{\beta} = \alpha \bar{\phi}^\alpha K^{\alpha - 1} N^{1 - \alpha} + (1 - \delta) \bar{\phi}
\]

\[
\zeta N^\gamma = (1 - \alpha) \bar{\phi}^\alpha K^\alpha N^{-\alpha}
\]

\[
Y = \bar{\phi}^\alpha K^\alpha N^{1 - \alpha}
\]

Change in the mean return

\[
\Delta \ln \bar{\phi} = -0.004 \quad \Rightarrow \quad (\Delta \ln Y)^{\text{Mean}} = -0.07
\]

Change in the risk premium ($\eta = 10$)

\[
\Delta \ln \bar{\phi} \text{ (risk-adjusted)} = -0.008 \quad \Rightarrow \quad (\Delta \ln Y)^{\text{Mean+Risk}} = -0.13
\]
Why are the long run effects so large?

Approximate actual data with

- $\hat{g}_{2007} \approx \{1, 1, \ldots, 1\}$ (53 observations)
- $\hat{g}_{2009} \approx \{1, 1, \ldots, 1, 0.94, 0.84\}$ (55 observations)

Mean

- $\bar{\phi}_{2009} = \frac{53 + 0.94 + 0.84}{53 + 1 + 1} = 0.996$

Risk adjustment

- Approximate SDF using weights $q(\phi) = \left(\frac{\phi}{\bar{\phi}}\right)^{-0.5\eta}$
- $\bar{\phi}_{2009} = \frac{53(0.98) + 0.94(1.33) + 0.84(2.34)}{53(0.98) + 1.33 + 2.34} = 0.992$
• Approximation: $\phi_t = \bar{\phi}_{2009}$, $lev_t = 0.7$, $\hat{G}_t = \hat{G}_{2009}$ $\forall t \geq 2009$
The firm’s problem

Two features simplify the analysis

- All shocks are iid
- No financing constraints

⇒ All firms solve

\[
\max_{\hat{k}_{t+1}, l_{t+1}, lev_{t+1}} \quad - \quad \hat{k}_{t+1} - \chi \mathcal{W}_t l_{t+1}
\]

\[+ \quad \mathbb{E}_t [M_{t+1} \Pi_{t+1}] \quad + \quad (\chi - 1) q_t \cdot lev_{t+1} \cdot \hat{k}_{t+1} \quad - \quad (1 - \theta) \mathbb{E}_t [M_{t+1} (1 - r_{t+1}) \Pi_{t+1}]\]

\text{Output + Undep capital} \quad + \quad \text{Tax advantage of debt} \quad - \quad \text{Cost of default}
The firm’s problem

\[ V (\Pi_{it}, B_{it}, S_t) = \max \left[ 0, \max_{d_{it}, \hat{k}_{it+1}, b_{it+1}, w_{it+1}, l_{it+1}} d_{it} + \mathbb{E}_t M_{t+1} V (\Pi_{it+1}, B_{it+1}, S_{t+1}) \right] \]

Dividends: \( d_{it} \leq \Pi_{it} - B_{it} - \hat{k}_{it+1} + \chi q_{it} b_{it+1} \)

Discounted wages: \( \mathcal{W}_t \leq w_{it+1} q \left( \hat{k}_{it+1}, l_{it+1}, B_{it+1}, S_t \right) \)

Future obligations: \( B_{it+1} = b_{it+1} + w_{it+1} l_{it+1} \)

Resources: \( \Pi_{it+1} = v_{it+1} \left[ A(\phi_{t+1} \hat{k}_{it+1})^\alpha l_{it+1}^{1-\alpha} + (1 - \delta) \phi_{t+1} \hat{k}_{it+1} \right] \)

Bond price: \( q \left( \hat{k}_{it+1}, l_{it+1}, B_{it+1}, S_t \right) = \mathbb{E}_t M_{t+1} \left[ r_{it+1} + (1 - r_{it+1}) \frac{\theta \tilde{V}_{it+1}}{B_{it+1}} \right] \)

- Dividends \( d_{it} \) can be negative, i.e. no financing constraints
- Default policy \( r_{it+1} \in \{0, 1\} \) and value \( \tilde{V}_{it+1} \equiv V (\Pi_{it}, 0, S_t) \)
- Aggregate state: \( S_t \) (includes information)
Optimality conditions

\[(1 - \theta) \mathbb{E}_t [M_{t+1}vf(v)] = \left(\frac{\chi - 1}{\chi}\right) \mathbb{E}_t [M_{t+1} (1 - F(v))] \]

\[1 = \mathbb{E}_t \left[ M_{t+1} R^k_{t+1} J^k(v) \right] - \chi \mathcal{W}_t \frac{l_{t+1}}{\hat{k}_{t+1}} \]

\[\chi \mathcal{W}_t = \mathbb{E}_t \left[ M_{t+1} (1 - \alpha) A \phi_{t+1}^\alpha \left( \frac{\hat{k}_{t+1}}{l_{t+1}} \right)^\alpha J'(v) \right] \]

where

\[R^k_{t+1} = \frac{A \phi_{t+1}^\alpha \hat{k}_{t+1}^{\alpha l_{t+1}^{1-\alpha}} + (1 - \delta) \phi_{t+1} \hat{k}_{t+1}}{\hat{k}_{t+1}} \]

\[J^k(v) = 1 + v(\chi - 1)(1 - F(v)) + (\theta \chi - 1) h(v) \]

\[J'(v) = 1 + h(v)(\theta \chi - 1) - v^2 f(v) \chi (\theta - 1) \]

Now,

\[\chi = 1 \quad \Rightarrow \quad v = 0 \quad \Rightarrow \quad J^k = J' = 1\]
Effect of post-2009 shocks

- **Shocks**
  - 2010: -0.2
  - 2020: 0
  - 2030: 0.1
  - 2040: 0

- **GDP**
  - Learning
  - No learning
  - Data

- **Investment**
  - 2010: -0.2
  - 2020: -0.1
  - 2030: 0
  - 2040: 0.1

- **Labor**
  - 2010: -0.2
  - 2020: -0.1
  - 2030: 0
  - 2040: 0.1
Other evidence: Business cycle accounting

- Data: Increase in capital/labor wedges post-2008
  - Karabarbounis (2011), Hall (2014)
- Model: Capital and labor wedge rise
  - By about 1% and 12%, (about 1/3rd higher than pre-crisis).