

HETEROGENEOUS INFORMATION AND LABOR MARKET FLUCTUATIONS*

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Abstract

Idiosyncratic productivity shocks induce larger adjustments to hiring than aggregate shocks, because general equilibrium effects on search frictions and wages partially offset the latter. When firms cannot disentangle the two shocks, they attribute aggregate disturbances partly to idiosyncratic factors and to that extent, respond more aggressively. This translates into increased aggregate volatility, an order of magnitude higher than the benchmark full information model. A calibrated model predicts moments that match closely levels observed in the US data. These results hold under both random and directed search specifications.

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1 Introduction

A growing body of work in macroeconomics examines the role of informational frictions in explaining cyclical fluctuations in economic activity¹. In this paper, I show that the inability of firms to disentangle aggregate shocks from idiosyncratic disturbances can generate fluctuations in aggregate labor market volatility that are an order of magnitude higher than under perfect information. This finding offers a resolution of a well-known puzzle. Shimer (2005) showed that a calibrated Mortensen and Pissarides (1994) model significantly underpredicts fluctuations in labor market activity, compared to empirically observed levels². I show that informational frictions completely eliminate this discrepancy between the data and model-predicted movements. In a calibrated model, informational frictions raise the standard deviation of aggregate employment growth (relative to that of output) from almost zero under full information to 0.57 (the corresponding moment in the post-war US data is 0.65). A similar amplification also obtains for other labor market variables such as vacancies and measures of labor market tightness. These results are robust to assumptions - both about the labor market (e.g. random versus directed search) as well as information (e.g. lagged arrival of information).

To illustrate the mechanism, consider a positive aggregate productivity shock. This increases economy-wide hiring activity, tightening the labor market and making it harder for the firm to fill vacancies. On the other hand, if the shock is specific to the firm, aggregate labor market conditions are unchanged. As a result, the firm will respond more aggressively to an idiosyncratic shock to its productivity than a shock of the same size to aggregate productivity. Now, suppose the information available to the firm does not allow it to fully disentangle the two sources of uncertainty³. In particular, the firm only sees its own productivity, a combination of aggregate and idiosyncratic components. Then, the firm attributes all changes in productivity, including ones

¹This literature dates back at least to Phelps (1970) and Lucas (1972). Recent papers have examined the role of strategic interactions, as in Woodford (2003) and Mankiw and Reis (2002), limited capacity for processing information, as in Mackowiak and Wiederholt (2009), market-generated information, as in Hellwig and Venkateswaran (2009) and Graham and Wright (2010) and real shocks, as in Mackowiak and Wiederholt (2010), Lorenzoni (2009) and Angeletos and La'o (2010a).

²See Mortensen and Nagypal (2007) for a recent survey of the extensive body of work on this issue.

³This is a natural assumption when reliable information about aggregates arrives only with a lag, forcing firms to use information generated in the normal course of their market activities to form forecasts about market conditions. Another justification for this assumption comes from limits to firms' information processing capacity (e.g. as in Sims, 2003). Mackowiak and Wiederholt (2009) show that, under such a constraint, firms choose to pay very little attention to direct signals of aggregate conditions. Since aggregate shocks are an order of magnitude smaller than idiosyncratic factors, firms have little incentive to devote costly resources to learning about them.

arising from the aggregate component, partly to idiosyncratic factors. This misattribution leads to an increased sensitivity of economy-wide hiring activity (and therefore, aggregate employment) to aggregate shocks.

The quantitative significance of this mechanism is supported by a well-documented empirical fact - fluctuations in employment at the firm-level are large relative to changes in aggregate hiring. This points to idiosyncratic factors that are significantly more volatile than aggregate conditions. In such an environment, the solution to the firms' filtering problem leads them to attribute changes in productivity almost entirely to idiosyncratic factors. To put it differently, the relative size of aggregate shocks makes learning about them from firm-level signals particularly difficult. This severity of firms' confusion translates into a quantitatively significant amplification in aggregate volatility⁴.

The paper develops a flexible and tractable solution method, by combining standard perturbation techniques for solving dynamic optimization problems with the approach for handling dynamic learning problems developed in Hellwig (2002). The result is a solution algorithm which can handle a large number of state variables - both fundamental and informational. A key assumption that makes this possible is that all shocks are revealed with a finite lag. In addition to being an intuitive way of capturing the diffusion of information about aggregates, this assumption also converts the learning problem into a finite-dimensional filtering problem, allowing a recursive characterization of the state space. The second element of the paper's methodological contribution is a calibration strategy, which uses micro data to impose discipline on the information structure. Since the only sources of information available to firms are their own market transactions, data on firm-level employment and hiring activity provide direct evidence on information flows.

This paper is related to several branches of literature. There is a direct relationship with the literature on the role of heterogeneous information in business fluctuations⁵. In most of these papers, dispersed information dampens (and delays) the response of the economy to aggregate shocks. In contrast, I find an increased sensitivity of the heterogeneously informed economy to aggregate productivity shocks. The source of this difference lies in the information structure. Most

⁴In the benchmark calibration, firm-specific shocks are also slightly more persistent than aggregate shocks. Since hiring decisions are dynamic, this also contributes to the amplification arising from the confusion. However, this channel turns out to be unimportant quantitatively.

⁵In addition to the papers mentioned in the text, an inexhaustive list includes Moscarini (2004), Bacchetta and Wincoop (2006), Reis (2006), Adam (2007), Veldkamp and Wolfers (2007), Angeletos and Pavan (2007), Gorodnichenko (2008), Angeletos and La'O (2010b), Hellwig (2008b, 2008a) and Lorenzoni (2009, 2010).

of these papers ⁶ use signals which are noisy observations of fundamentals. Therefore, shocks to fundamentals are partly attributed to noise and to that extent, have no effect on responses. In this paper, on the other hand, signals are combinations of aggregate and idiosyncratic factors. The resulting confusion leads to an amplified response to the aggregate shock⁷.

There are several important exceptions. In Lucas (1972), agents cannot distinguish purely nominal shocks from island-specific demand shocks. More recently, Hellwig and Venkateswaran (2009) investigate the implications of signals which combine aggregate and idiosyncratic shocks in a nominal-price setting context. Other recent applications with similar information structures include Amador and Weill (2010) and Graham and Wright (2010). However, none of these papers focus on the role of search frictions, the source of the general equilibrium effects (and therefore, the amplification) in this paper.

It is well-known that standard calibrations of search models (e.g. Shimer2005) lead to relatively small fluctuations in the value of a vacancy for a firm. As a result, resources devoted to hiring - and consequently, employment - also do not fluctuate much over the cycle, contrary to what we see in the data. A number of papers propose modifications designed to make the value of a vacancy more responsive to shocks⁸. The informational frictions in this paper, on the other hand, cause the *perceived* value of a vacancy to be very volatile, even when the *actual* value behaves as in the standard model. To highlight the potential for amplification from this channel, I abstract from the other modifications and stick as close as possible to the standard model⁹.

Finally, I draw calibration targets from the literature on empirical properties of firm-level employment and hiring activity. The firm dynamics literature¹⁰ documents employment growth rates

⁶For example, see the benchmark environments of Mackowiak and Wiederholt (2010) and Angeletos and La'o (2010a). Lorenzoni (2009) has agent-specific productivity shocks but the iid assumption implies that they have no effect on forward looking pricing decisions and therefore, act very much like observational noise.

⁷Li and Weinberg (2003) employ a similar confusion to generate differences in the cyclical response of investments at small and large firms. Small firms also tend to be younger and therefore, are less informed about their true productivity.

⁸For example, Hall (2005) makes wages 'sticky', leading to bigger fluctuations in the value appropriated by the firm out of a match. Pissarides (2009) introduces, as does Cheremukhin (2010), fixed costs associated with hiring. Hagedorn and Manovskii (2008) pursue an alternative calibration strategy which reduces the *level* of the match surplus, but leads to larger *percentage* fluctuations. The main contribution of this paper is to introduce a novel channel of amplification, distinct from the mechanisms discussed above.

⁹Moreover, since my focus here is the job creation margin, I also abstract from other margins of adjustment, e.g. endogenous job destruction (as in Cheremukhin, 2010, den Haan and Ramey, 2000, Mortensen and Nagypal, 2007 or Eyiungor, 2010) or on-the-job search (as in Menzio and Shi, 2011).

¹⁰See Davis et al. (2007) or Franco and Philippon (2007).

at the firm level that are significantly more variable than aggregate employment and are independent of firm size. These findings help pin down the stochastic properties of firm-specific productivity processes in my model. The model also has idiosyncratic shocks to the efficiency of a firm's recruiting process, which are calibrated to match targets from recent work using firm-level vacancy data by Davis, Faberman and Haltiwanger (2010).

The rest of this paper proceeds as follows. Section 2 presents a simple 2-period example which highlights the basic economic forces at work and highlights the potential for amplification in this mechanism. Section 3 lays out the full model and defines the relevant equilibrium concepts. Sections 4 and 5 describe the solution algorithm and the calibration strategy respectively. Section 6 discusses the numerical results, along with some robustness exercises. Section 7 extends the analysis to random search and Nash bargaining and section 8 contains a brief conclusion.

2 A simple example

In this section, I lay out a simple example, which will highlight the basic economic mechanism at work in the full model. A 2-period economy is populated by a unit measure of both firms and workers. Let N_{it} , $t = 1, 2$ denote the mass of employed workers at firm i in period t . At the beginning of period 1, workers and firms are uniformly matched i.e. $N_{i1} = 1 \forall i$. Period 1 output is produced according to:

$$Y_{i1} = e^{a_1 + a_{i1}} (N_{i1})^\alpha = e^{a_1 + a_{i1}} ,$$

where $e^{a_1 + a_{i1}}$ represents the firm's productivity in period 1. Productivity has an aggregate and an idiosyncratic component, denoted a_1 and a_{i1} respectively. Both are mean-zero, normally distributed random variables

$$\begin{aligned} a_1 &\sim N(0, \sigma^2) , \\ a_{i1} &\sim N(0, \hat{\sigma}^2). \end{aligned}$$

I assume that a law of large numbers applies¹¹ to the cross-sectional distribution of a_{i1} i.e.

$$\int a_{i1} di = 0.$$

¹¹I maintain this assumption about idiosyncratic shocks throughout the paper. See Sun (2006) for the precise construction of a probability space where the exact law of large numbers holds for a continuum of pairwise independent random variables.

Output in period 2 is produced according to the same decreasing returns-to-scale technology

$$Y_{i2} = e^{a_2 + a_{i2}} (N_{i2})^\alpha ,$$

where a_2 and a_{i2} are the aggregate and idiosyncratic components of period 2 productivity. They are linked to their period 1 counterparts as follows:

$$\begin{aligned} a_2 &= \rho a_1 & \rho &\in [0, 1] , \\ a_{i2} &= \hat{\rho} a_{i1} , & \hat{\rho} &\in [0, 1] . \end{aligned}$$

The commonly-known parameters ρ and $\hat{\rho}$ index the persistence of the aggregate and idiosyncratic components respectively¹².

After period 1 production, the firm's current matches are exogenously destroyed. In other words, to be able to produce in period 2, the firm has to go through a hiring process, which is hampered by search frictions. Two commonly-used specifications of these frictions are random matching and directed search. In the former, all firms in the economy post vacancies in a single labor market and are matched with potential hires in accordance with an aggregate matching function. Under directed search, firms post wages and vacancies in different 'submarkets' and hire from the pool of workers who choose to search in that submarket. I consider these two approaches separately.

2.1 Random Matching

The matching process is described by an aggregate matching function, which takes as inputs the mass of unemployed (denoted U) and the total number of vacancies posted in the economy (denoted V). The total mass of successful matches is given by:

$$M = \zeta f(U, V) = \zeta U^\eta V^{1-\eta} = \zeta V^{1-\eta} , \quad \eta \in (0, 1) ,$$

where η is the elasticity of the matching function with respect to the measure of unemployed¹³. The last equality follows from the assumption that all matches are destroyed in first period, leading to $U = 1$. Each vacancy is filled with the same probability, $\frac{M}{V} = \zeta V^{-\eta}$. The parameter η thus also indexes the elasticity of the vacancy-filling rate with respect to market tightness (or equivalently, the total mass of vacancies posted).

¹²Introducing additional shocks to period 2 productivities - whether aggregate or idiosyncratic - will not change any of the results in this section. Therefore, I abstract from any additional uncertainty.

¹³I assume that ζ and the variances of the shocks are such that the condition $M \leq \min(U, V)$ holds with an arbitrarily high probability.

Given this specification of the matching technology, firm i 's employment in period 2 is given by

$$N_{i2} = V_i \zeta V^{-\eta},$$

where V_i is the measure of vacancies it posts. The (constant) marginal cost of posting a vacancy is a unit of output.

In both periods, wages are determined by a simple bargaining game. In the first period, the firm holds all the bargaining power and appropriates the entire output¹⁴. In period 2, with probability ϑ , the firm gets to make a take-it-or-leave-it offer to the workers and with the complementary probability, the workers make an offer to the firm. Dividends in the two periods are therefore,

$$\begin{aligned} D_{i1} &= Y_{i1} - V_i = e^{a_1+a_{i1}} - V_i \\ D_{i2} &= Y_{i2} = e^{a_2+a_{i2}}(N_{i2})^\alpha \quad \text{with probability } \vartheta \\ &= 0 \quad \text{with probability } 1 - \vartheta \end{aligned}$$

The Firm's Problem: Firms maximize the expected value of (undiscounted) dividends. Formally, firm i solves

$$\max_{V_i} e^{a_1+a_{i1}} - V_i + \vartheta \mathbb{E}_i e^{a_2+a_{i2}}(N_{i2})^\alpha$$

where

$$N_{i2} = \zeta V^{-\eta} V_i$$

and the operator, \mathbb{E}_i , denotes expectation conditional on firm i 's information.

2.1.1 Optimality

The first-order condition of the firm's problem is

$$1 = \underbrace{\zeta V^{-\eta}}_{\text{Probability}} \underbrace{\vartheta}_{\text{Firm's share}} \underbrace{\alpha \mathbb{E} \left(e^{a_2+a_{i2}} \right) (N_{i2})^{\alpha-1}}_{\text{Match surplus}}. \quad (1)$$

The left-hand side of (1) is the marginal cost of a vacancy and the right side is the expected marginal value from posting a vacancy. Assuming conditional log-normality (which will be shown to hold), substituting for second period employment and taking logs on both sides, this can be written as

$$(1 - \alpha)v_i = \mathbb{E}_i(a_2 + a_{i2}) - \alpha\eta\mathbb{E}_i(v) + \text{Const}. \quad (2)$$

¹⁴This is not crucial. The results in this section are unaffected by alternative assumptions about how the surplus is split in period 1.

where $v \equiv \int v_i di$ and the constant term is a function of parameters and the (commonly known) second moments of random variables on the right hand side. Equation (2) is intuitive - the firm's hiring decision depends positively on its expected productivity and negatively on the aggregate number of vacancies posted in the economy. The latter effect arises because the vacancy yield of the firm (i.e. the number of new hires each vacancy generates) decreases with economy-wide hiring activity. As a result, for a given expectation of fundamentals (i.e. future productivity), the number of vacancies posted by a firm is decreasing in its belief about aggregate hiring activity. The greater the aggregate number of vacancies posted in the economy, the lower is the incentive of an individual firm to post vacancies. In the language of the heterogeneous information literature, this feature makes hiring decisions strategic substitutes. The effect of this interaction is greater (i.e. more negative), the larger the elasticity of the vacancy yield with respect to aggregate vacancies (i.e. η).

The firm's belief about v takes the form of a conjecture about its relationship with a_1 , the only source of aggregate uncertainty in the economy. In a rational expectations equilibrium, this conjecture is the same as the true relationship. I guess (and verify) that this relationship is linear (up to a constant):

$$v = \phi a_1, \quad (3)$$

where ϕ is an endogenous coefficient. Substituting this conjecture and the laws of motion for productivity in (2),

$$(1 - \alpha)v_i = \rho \mathbb{E}_i(a_1) + \hat{\rho} \mathbb{E}_i(a_{i1}) - \alpha\eta\phi \mathbb{E}_i(a_1) + \text{Const.} \quad (4)$$

$$= (\rho - \alpha\eta\phi) \mathbb{E}_i(a_1) + \hat{\rho} \mathbb{E}_i(a_{i1}) + \text{Const.} \quad (5)$$

Thus, the firm's optimal response to a perceived aggregate shock, a_1 , is dampened by the effects of labor market congestion (captured by the $-\alpha\eta\phi$ in the first term). The firm-specific component of productivity does not affect aggregate labor market conditions and therefore, evokes a response that depends only on its persistence.

2.1.2 Full Information

A natural benchmark is the case where firms are assumed to have full information about the nature of shocks i.e. they observe both components of their productivity shock separately. Therefore,

$$\mathbb{E}_i(a_1) = a_1 \quad \mathbb{E}_i(a_{i1}) = a_{i1}.$$

Substituting in (5), integrating over all i and ignoring the constant terms

$$\begin{aligned} (1 - \alpha) \int v_i di &= (\rho - \alpha\eta\phi) \int \mathbb{E}_i(a_1) di + \hat{\rho} \int \mathbb{E}_i(a_{i1}) di \\ \Rightarrow (1 - \alpha) v &= (\rho - \alpha\eta\phi)a_1 + \hat{\rho} \int a_{i1} di = (\rho - \alpha\eta\phi) a_1. \end{aligned}$$

Thus, under full information, the firm's conjecture (3) is verified when

$$(1 - \alpha) \phi^{\text{Full}} = (\rho - \alpha\eta \phi^{\text{Full}}) \Rightarrow \phi^{\text{Full}} = \frac{\rho}{1 - \alpha + \alpha\eta}. \quad (6)$$

2.1.3 Dispersed Information

Now, assume that, at the time of making its hiring decision, each firm only observes its composite productivity $a_1 + a_{i1}$. A direct application of Bayes' rule leads to

$$\mathbb{E}_i(a_1) = \pi(a_1 + a_{i1}) \quad \mathbb{E}_i(a_{i1}) = (1 - \pi)(a_1 + a_{i1}), \text{ where } \pi = \frac{\sigma^2}{\sigma^2 + \hat{\sigma}^2}.$$

Thus, the signal is attributed to the 2 components according to the ratio of their variances. Substituting in (5) and integrating over all i ,

$$\begin{aligned} (1 - \alpha) \int v_i di &= (\rho - \alpha\eta\phi) \int \mathbb{E}_i(a_1) di + \hat{\rho} \int \mathbb{E}_i(a_{i1}) di \\ (1 - \alpha) v &= (\rho - \alpha\eta\phi)\pi \int (a_1 + a_{i1}) di + \hat{\rho}(1 - \pi) \int (a_1 + a_{i1}) di \\ \Rightarrow (1 - \alpha) v &= (\rho - \alpha\eta\phi)\pi a_1 + \hat{\rho}(1 - \pi) a_1, \end{aligned}$$

where the last equality makes uses of the fact that $\int a_{i1} di = 0$. Under heterogeneous information, the firms' conjecture (3) is verified when

$$(1 - \alpha) \phi^{\text{Het}} = (\rho - \alpha\eta \phi^{\text{Het}})\pi + \hat{\rho}(1 - \pi) \quad (7)$$

$$\Rightarrow \phi^{\text{Het}} = \frac{\pi \rho + (1 - \pi) \hat{\rho}}{1 - \alpha + \alpha\eta\pi}. \quad (8)$$

A comparison of (6) and (8) reveals that heterogeneous information changes the equilibrium response of aggregate hiring in 2 ways. The first comes from changes in the forecast of future productivity in response to an aggregate shock. Under full information, firms observe a_1 perfectly and correctly forecast its implications for their productivity next period. Under heterogeneous information, firms attribute a fraction $1 - \pi$ of all changes in their signals (including those coming from aggregate shocks) to an idiosyncratic shock. Therefore, the firm's forecast for period 2 productivity is computed using a weighted average of the persistence of the 2 components, with weights determined by π . This is reflected in the numerator in (8). Whether this leads to an increased

responsiveness of vacancies depends on the relative persistence of the two shocks. If $\hat{\rho}$ is greater (resp. less) than ρ , then the aggregate shock is misattributed to a more (less) persistent shock and therefore, induces a larger (smaller) response.

The second, and more important, change arises because informational frictions also affect the firm's expectation of aggregate labor market conditions (through the $\mathbb{E}_i(v)$ term). For example, a positive aggregate shock increases overall hiring activity and therefore, leads to lower vacancy yields. However, to the extent aggregate shocks are mistakenly attributed to idiosyncratic factors, firms underestimate this effect. Since their hiring decisions depend negatively on expectations of overall market tightness, this underestimation works to increase the response of vacancies posted. This effect shows up through the π multiplying the $\alpha\eta$ term in the denominator of (8).

The following proposition shows that these two effects lead to an amplified response in aggregate hiring activity under dispersed information, provided the idiosyncratic shock is sufficiently persistent.

Proposition 1 *Suppose $\hat{\rho} \geq \rho \left(\frac{1-\alpha}{1-\alpha+\alpha\eta} \right)$. Then, the economy under heterogeneous information is more responsive to aggregate shocks i.e. $\phi^{Het} \geq \phi^{Full}$.*

Note that $\left(\frac{1-\alpha}{1-\alpha+\alpha\eta} \right) < 1$, so informational frictions can lead to amplification even when idiosyncratic shocks are less persistent than aggregate shocks. This is precisely because of the second effect discussed above (i.e. through firms' expectations about aggregate labor market conditions). To see this, consider the case as the strategic interactions disappear i.e. $\eta \rightarrow 0$. Then,

$$\phi^{Full} \rightarrow \frac{\rho}{1-\alpha} \qquad \phi^{Het} \rightarrow \frac{\pi \rho + (1-\pi) \hat{\rho}}{1-\alpha}.$$

In other words, without the equilibrium effects introduced by the aggregate matching function, the differences in responses under full and heterogeneous information arise purely from the relative persistence of the two shocks. In this limiting case, heterogeneous information leads to amplification in aggregate responsiveness if and only if idiosyncratic shocks are more persistent than aggregate ones. More generally, the greater the link between vacancy yield and aggregate hiring, i.e. the higher the value of η , the lower is the threshold level of idiosyncratic persistence necessary to generate amplification.

2.2 Directed Search

Next, I will show that the above amplification result extends to a directed search environment where firms can post higher wages to attract workers. The underlying intuition is very similar to the

random search case - under dispersed information, firms attribute aggregate shocks to idiosyncratic factors and underestimate the changes in overall labor market conditions (in particular, the market utility of workers).

The setup is in the spirit of the competitive search literature (e.g. Moen, 1997, Acemoglu and Shimer, 1999, Guerrieri 2008, Menzio and Shi 2011). Firms post both vacancies and wages, recognizing that their posted wages affect the probability of filling vacancies. In equilibrium, each level of wage is associated with an (endogenously determined) market tightness, a relationship firms take as given when making their optimal choices. Given a choice (W_i, V_i) by firm i , its employment in period 2 is

$$N_{i2} = V_i \zeta \Theta_i^{-\eta},$$

where Θ_i is the level of tightness associated with the wage W_i and ζ is a parameter. In other words, $\zeta \Theta_i^{-\eta}$ is the probability that a vacancy with a wage W_i will lead to a successful match.

For simplicity, I assume that workers perfectly observe¹⁵ the entire cross-section of submarkets, each identified by a wage and tightness pair (W, Θ) and choose one to conduct search in. A worker searching in a market with tightness Θ finds a job with probability $\zeta \Theta^{1-\eta}$. Since workers are all identical, any submarket that attracts workers must yield the same expected utility to a worker. Denote this (endogenous) utility by S . Then, in any active submarket j , we must have

$$W_j \zeta \Theta_j^{1-\eta} = S. \tag{9}$$

Or in logs,

$$w_j + (1 - \eta)\theta_j + \text{Constant} = s. \tag{10}$$

The market tightness in any submarket is defined as the ratio of the measure of vacancies posted to that of workers searching for jobs in that submarket, i.e.

$$\Theta_j = \frac{V_j}{U_j}$$

Substituting for Θ_j in (10),

$$w_j + (1 - \eta)v_j - (1 - \eta)u_j + \text{Constant} = s.$$

Integrating over all active submarkets,

$$w + (1 - \eta)v - (1 - \eta)\bar{u} + \text{Constant} = s$$

¹⁵In the full model, I relax this assumption.

where w and v are the average wages and vacancies posted in the economy. Substituting in (10) yields this relationship between the posted wage and the associated market tightness in any active submarket:

$$(1 - \eta)\theta_j = w + (1 - \eta)v - w_j + \text{Constant}. \quad (11)$$

where I make use of the fact that \bar{u} is also a constant. The firm takes this relationship between wages and market tightness (and through that, the probability of filling a vacancy) as given¹⁶.

Formally, the firm's problem is

$$\max_{V_i, W_i} e^{a_1+a_{i1}} - V_i + \mathbb{E}_i e^{a_2+a_{i2}}(N_{i2})^\alpha - w \cdot N_{i2}$$

subject to

$$N_{i2} = V_i \zeta \theta_i^{-\eta}.$$

where θ_i is determined by (11).

The optimality conditions, in logs, are given by:

$$w_i = \eta \mathbb{E}_i (w + (1 - \eta)v) + \text{Constant}. \quad (12)$$

$$(1 - \alpha)v_i = \mathbb{E}_i(a_2 + a_{i2}) - \alpha\eta \mathbb{E}_i (w + (1 - \eta)v) + \text{Constant}. \quad (13)$$

Thus, the economy-wide wage and vacancy decisions affect the market utility of unemployed workers, which in turn determines a firm's ability to hire at a given wage. These equilibrium effects play the same role as they did in the random search case - they serve to dampen the firm's desire to post vacancies in response to aggregate shocks, but have no effect on the response to idiosyncratic ones.

The solution strategy follows the same guess-and-verify approach we used for the random search case. The key variable of interest is the response of the total number of vacancies to the aggregate productivity shock. These response coefficients, under full and dispersed information, are:

$$\begin{aligned} \phi_v^{\text{Full}} &= \frac{(1 - \eta)\rho}{(1 - \alpha)(1 - \eta) + \alpha\eta} \\ \phi_v^{\text{Het}} &= \frac{(1 - \eta\pi)(\rho\pi + (1 - \pi)\hat{\rho})}{(1 - \alpha)(1 - \eta\pi) + \alpha\eta\pi} \end{aligned}$$

where $\pi = \frac{\sigma^2}{\sigma^2 + \hat{\sigma}^2}$ once again denotes the fraction of the signal attributed to the aggregate shock. As before, it enters the response coefficient in 2 ways. One, it changes the expected persistence

¹⁶Note that the firm uses this to forecast market tightness in *all* submarkets, even those that might not be active in equilibrium. This is a common assumption in the literature and is usually justified using a 'tremble' argument, e.g. by assuming that an arbitrarily small number of firms post vacancies in every market.

of the shock, to the extent that aggregate and idiosyncratic shocks differ in their autocorrelation. Two, when aggregate shocks are misattributed to idiosyncratic factors, firms underestimate the change in workers' utilities and therefore, in the market tightness associated with any given level of wages. This second effect is reflected in the fact that all the instances of η in the expression for ϕ_v^{Het} are multiplied by π .

The next result shows that Proposition 1 extends to this environment as well - provided idiosyncratic shocks are not too transitory, the economy under heterogeneous information displays a greater sensitivity of hiring to aggregate productivity shocks.

Proposition 2 *Suppose $\hat{\rho} \geq \rho \left(\frac{1-\alpha}{1-\alpha+\alpha\eta} \right)$. Then, total vacancies are more responsive to aggregate shocks in the economy under heterogeneous information i.e. $\phi_v^{\text{Het}} \geq \phi_v^{\text{Full}}$.*

Note that, as in the random search case, the mechanism does not depend on idiosyncratic shocks being more persistent than aggregate shocks. The greater the strength of the general equilibrium interactions, parameterized by η , the lower the degree of persistence in firm-specific uncertainty required for amplification.

2.3 Relation to the Literature

Equation 1 provides an intuitive way to understand both the source of the volatility puzzle highlighted by Shimer (2005) and the mechanism through which informational frictions operate. The value of a vacancy can be decomposed into 3 parts:

$$\text{Cost of a vacancy} = \mathbb{E}_i [\text{Probability of a match} \times (\text{Firm's share} \times \text{Match surplus})] \quad (14)$$

Suppose the left hand side is well-approximated by a constant. Given a matching function and data on employment/vacancies, cyclical movements in the probability of a match can be directly measured in the data. Under standard assumptions, this probability is very volatile i.e. declines (rises) sharply during booms (recessions). Then, for this equation to hold, the term within parentheses i.e. the benefit to the firm from a successful match must rise (decline) sharply during booms (recessions). In other words, a model can match the observed fluctuations in employment/vacancies data only by making the benefit to the firm from a successful match vary strongly with the cycle. This is the source of the puzzle - the standard search model does not generate sufficient cyclical movement in this object and therefore, predicts modest fluctuations in labor market activity. Various modifications to the standard model have been proposed to make the firm's value from a match more responsive to shocks. For example, when wages are sticky, as in Hall (2005), the firm's share

becomes more cyclical. Training costs, as in Pissarides (2009), or higher outside options of workers, as in Hagedorn and Mannovskii (2008), also generate bigger fluctuations in the net surplus from a match. In the presence of information frictions, however, firms equate the cost of a vacancy to the *perceived* benefit from posting one. When firms mistake an aggregate shock for an idiosyncratic one, they underestimate the movements in the probability of a match and overestimate the change in the other terms. This mechanism acts basically through the \mathbb{E}_i operator - a very different channel from the modifications discussed above. As a result, it is present even when the *actual* benefit from a vacancy behaves as in the standard model.

2.4 Testing the theory

I conclude this section with a brief discussion on empirical evidence in support of the theory. Obviously, micro evidence on information/beliefs would be the most direct way to test the information friction at work here, but reliable data on that is hard to come by. However, there are other, more indirect, ways to test the theory. The analysis above highlights the role of two key parameters - the volatility of idiosyncratic uncertainty (or more precisely, the relative volatility π) and the strength of general equilibrium linkages (captured by η in the simple model). In particular, the greater the relative variance of the idiosyncratic shocks (i.e. the lower is $\frac{\sigma^2}{\sigma^2}$), the more severe the confusion (i.e. the lower is π) and therefore, the greater the sensitivity of aggregate employment to shocks. Similarly, stronger the equilibrium linkages η , the greater is the potential for amplification.

This points directly to a couple of testable implications. First, informational frictions are a source of systematic positive correlation between idiosyncratic and aggregate volatility¹⁷ Depending on the interpretation of the model, this could be tested using cross-country, sectoral or time series data. Countries (or sectors) facing higher uncertainty at the micro level should also display a heightened sensitivity to aggregate (or sectoral) shocks. For example, the firm dynamics literature has documented that smaller and younger firms are subject to larger idiosyncratic shocks affecting¹⁸. The model implies that they should also display a greater responsiveness to aggregate shocks. This is the subject of a long-standing debate in the literature. Davis et. al. (1998) show that the time series volatility of total employment at young firms is higher than that of aggregate employment. Fort, Haltiwanger, Jarmin and Miranda (2012) also find significant differences in the behavior of small and large firms, but emphasize the importance of firm age. Moscarini and Postel-

¹⁷Needless to say, this is not the only theory capable of generating this correlation. See Campbell and Fisher (2004) for an alternative explanation.

¹⁸See, for example, Davis, Haltiwanger and Schuh (1998).

Vinay (2012), however, find contrary evidence using filtered data. In a time series interpretation, the model implies that a declining trend in idiosyncratic variability should be accompanied by a moderation of aggregate volatility, again an observation which has some empirical support¹⁹. A second testable implication is that countries/regions with more integrated labor markets will be more responsive to aggregate shocks. Recent work indicates that this might be a promising avenue²⁰, though the obvious empirical challenge here is to construct a reliable and consistent measure of labor market integration.

3 The Full Model

In this section, I lay out a micro-founded, dynamic version of the simple model studied in the previous section. The environment is similar to that used in the search and matching literature, modified to allow for heterogeneity and dispersed information. The full model will have a number of features - infinite horizon, a more standard specification of technology and recruiting, a richer set of signals - that were not present in the simple example discussed above. These features preclude an analytical characterization of the solution but allow a more robust quantitative evaluation of the role of informational frictions. They will also present a few challenges in solving the model - a dynamic learning problem, both physical and informational state variables, to cite a couple. Addressing these challenges is part of the methodological contribution of this paper and I discuss them in greater detail in Section 4.

For my baseline analysis, I use directed search and wage-posting. In Section 7, I show that the results hold - both qualitatively and quantitatively - under a random search specification.

3.1 Preferences and Technology

I denote the history of the economy up to time t by s^t . The state space for s^t includes all aggregate and idiosyncratic shocks.

Households: There is a representative household with a measure 1 of worker-members. The

¹⁹Davis et. al. (2007) document this for the US during the period from 1980-2000, a period also marked by lower aggregate volatility.

²⁰Amaral and Tasci (2012) analyze labor market volatility across OECD countries and find significant cross-country variation. For example, the US, Canada, UK, Germany, Sweden and Norway show bigger fluctuations in unemployment rates (normalized by the volatility of productivity) compared to France, Japan, Hungary and Turkey.

household is risk-neutral and maximizes the expected discounted sum of utilities of all its members

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \Pi(s^{t+\tau}) C_{t+\tau}(s^{t+\tau}),$$

where $C_{t+\tau}(s^{t+\tau})$ is aggregate household consumption.

Firms: The firm maximizes the expected discounted sum of dividends:

$$\mathbb{E}_{it} \sum_{\tau=0}^{\infty} \beta^\tau D_{it+\tau},$$

subject to

$$D_{it+\tau} + K_{i,t+\tau+1} = Y_{it+\tau} - W_{it+\tau} N_{it+\tau} + K_{i,t+\tau}(1 - \delta) - \frac{\psi}{2} \left(\frac{I_{it+\tau}}{K_{it+\tau}} - \delta \right)^2 - V_{it,t+\tau},$$

where the explicit dependence on s^t is suppressed for brevity. The expectation \mathbb{E}_i is taken with respect to firm i 's information set (to be defined later).

The firm operates a Cobb-Douglas technology:

$$Y_{it} = A_t A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2}$$

where aggregate and idiosyncratic productivity shock processes are denoted by A_t and A_{it} respectively. I assume decreasing returns at the firm level i.e. $\alpha_1 + \alpha_2 < 1$.

Labor Markets: Since both firms and workers are risk-neutral and all match destruction is exogenous, the exact time path of promised wage payments is irrelevant. In period $t - 1$, the firm posts a measure $V_{i,t-1}$ of vacancies, each offering an expected discounted present value W_{it} to the worker over the life of the match. This results in a measure of new matches (and a new level of firm employment N_{it}) according to:

$$N_{i,t} = (1 - \delta_n) N_{i,t-1} + V_{i,t-1} \left(\bar{\mu} \Theta_{i,t-1}^{-\eta} M_{it} \right). \quad (15)$$

where δ_n is the (exogenous) separation rate, $\bar{\mu}$ is a parameter and $M_{i,t}$ is an (exogenous) idiosyncratic shock to matching efficiency²¹. The tightness of the submarket in which the firm posts wages is denoted by $\Theta_{i,t-1}$. As in the simple example, this is linked to the posted wage through the worker's indifference condition. Formally, all submarkets active at time $t - 1$ must deliver the same utility to the worker,

$$S_{t-1} = \bar{\mu} \Theta_{i,t-1}^{1-\eta} \mathbb{E}_{t-1} M_{it} S_{t-1}^e(W_{it-1}) + (1 - \bar{\mu} \Theta_{i,t-1}^{1-\eta} \mathbb{E}_{t-1} M_{it}) \beta \mathbb{E}_{t-1} S_t \quad (16)$$

²¹Cheremukhin and Restrepo-Echavarría (2010) find a significant role for aggregate shocks to matching efficiency in explaining the US post-war data. Here, I employ only idiosyncratic ones.

where

$$S_{t-1}^e(W_{it-1}) = W_{it-1} + \sum_{s=1}^{\infty} \beta^s (1 - \delta_n)^{s-1} \delta_n \mathbb{E}_{t-1} S_{t-1+s} \quad (17)$$

is the value of finding a job with a discounted present value W_{it} .

Letting $\tilde{S}_{t-1} \equiv \sum_{s=1}^{\infty} \beta^s (1 - \delta_n)^{s-1} \delta_n \mathbb{E}_{t-1} S_{t-1+s}$, we can write the worker's value function recursively:

$$S_{t-1} = \bar{\mu} \Theta_{i,t-1}^{1-\eta} \mathbb{E}_{t-1} M_{it} \left(W_{it-1} + \tilde{S}_{t-1} - \beta \mathbb{E}_{t-1} S_t \right) + \beta \mathbb{E}_{t-1} S_t \quad (18)$$

$$\tilde{S}_{t-1} = \beta \delta_n \mathbb{E}_{t-1} S_t + \beta (1 - \delta_n) \mathbb{E}_{t-1} \tilde{S}_t \quad (19)$$

This system of equations implicitly defines a relationship between wages and market tightness Θ_{it-1} . The firms takes this as given while making its wage and vacancy choices, i.e. it recognizes that given a vacancy posted at wage W_{it-1} has a probability $\bar{\mu} \Theta_{i,t-1}^{-\eta} M_{it}$ of being filled.

The expectation operator \mathbb{E}_{t-1} conditions on the information set of unemployed workers at time $t - 1$. If unemployed workers can perfectly observe the entire cross-section of posted wages in the economy, then the above system implies a one-to-one relationship between posted wages and market tightness (and therefore, vacancy filling rates). The presence of the idiosyncratic matching shock M_{it} in (15) introduces noise into this tight connection. One interpretation of this shock is that it represents informational frictions on the workers' side. In other words, idiosyncratic noise in the matching technology is a tractable way to model the errors made by imperfectly informed workers in choosing a submarket. An alternative interpretation is that recruiter efficiency at the firm level is subject to idiosyncratic disturbances. In section 5, I adopt a calibration strategy that uses recent evidence on firm-level vacancy filling rates and is valid under both these interpretations. The key role of these shocks in the model is to slow down learning about aggregate labor market conditions from the results of firm-level search efforts. To see this, note that without these shocks, a firm can perfectly infer the tightness of its submarket (and therefore, aggregate conditions) with a one-period lag.

3.2 Aggregate and Idiosyncratic Shock Processes

In this subsection, I specify the stochastic processes followed by the aggregate and idiosyncratic shocks. All shocks are assumed to follow AR(1) processes in logs, i.e.

$$\ln A_t = \rho \ln A_{t-1} + u_t \quad (20)$$

$$\ln A_{it} = \rho_a \ln A_{it-1} + u_{it}^a \quad (21)$$

$$\ln M_{it} = \rho_m \ln M_{it-1} + u_{it}^m. \quad (22)$$

where u_t , u_{it}^a and u_{it}^m are normally distributed with mean zero and variances σ^2 , σ_a^2 and σ_m^2 respectively.

I also adopt the standard convention that a law of large numbers applies to the cross-sectional distribution of idiosyncratic shocks i.e. $\forall t$

$$\int u_{it}^j di = 0, \quad j = a, m.$$

3.3 Information Structure

All shocks become common knowledge after a lag of T^* periods. This is a simple way to capture the lags in the arrival of information through published aggregate data and/or asset prices. Apart from this lagged information, firms do not observe any aggregates directly. They only have access to variables which arise in the natural course of their business - productivities and outcomes of their labor market activities. Firms use these signals along with knowledge of the underlying equilibrium relationships to form forecasts about current and future aggregate conditions. Formally, I assume that firm i 's information set at time t , denoted \mathcal{I}_{it}

- Productivities: $\{a_{t-\tau} + a_{i,t-\tau}\}_{\tau=0}^{\infty}$
- All firm-specific variables: $\{v_{i,t-\tau-1}, k_{i,t-\tau}, n_{i,t-\tau}, w_{i,t-\tau}\}_{\tau=0}^{\infty}$
- $\{u_{t-T^*-\tau}, u_{it-T^*-\tau}^a, u_{it-T^*-\tau}^m, \}_{\tau=1}^{\infty}$

3.4 Optimality

Writing the firm's problem recursively,

$$\begin{aligned} \mathbb{V}(N_{it}, K_{it}, \Omega) = & \max_{W_{it}, V_{it}, I_{it}} Y_{it} - V_{it} - I_{it} - \frac{\psi}{2} \left(\frac{I_{it}}{K_{it}} - \delta \right)^2 \\ & + \beta \mathbb{E}_{it} \mathbb{V}(N_{i,t+1}, K_{i,t+1}, \Omega') - \beta W_{it} V_{it} \mathbb{E}_{it} \left(\bar{\mu} \Theta_{it}^{-\eta} M_{it+1} \right) \end{aligned}$$

subject to

$$N_{i,t+1} = N_{it}(1 - \delta_n) + V_{it}\bar{\mu}\Theta_{it}^{-\eta}M_{it+1}$$

$$K_{i,t+1} = K_{it}(1 - \delta) + I_{it}$$

and the system of equations (18)-(19).

3.5 An Approximate Equilibrium

A rational expectations equilibrium for this economy is defined in the usual manner (see, for example, Townsend, 1983). Appendix A presents the formal equilibrium definition. Solving for the exact equilibrium in this economy, however, is quite challenging. With significant fundamental and informational heterogeneity, the distribution of types is a high dimensional state variable. Therefore, I construct and solve for an equilibrium in the neighborhood of a deterministic steady state (i.e. one without aggregate or idiosyncratic shocks). In this region, the equations characterizing individual decisions are well-approximated by a first order log-approximation. Further, log-deviations of aggregate variables are assumed to be well approximated by the cross-sectional averages of the log-deviations of individual state variables. Formally, an approximate equilibrium is a set of log-deviations of

- i. Aggregate variables $s_t, \tilde{s}_t, c_t, n_t, v_t$ as linear functions of (u_t, u_{t-1}, \dots)
- ii. Firm employment n_{it}
- iii. Firm decisions $k_{it+1}, w_{it}, v_{it}, d_{it}$ as linear functions of the (log-deviations) of elements in the firm's information set

such that, to a first-order approximation,

- The choices in (iii) solve the firm's problem, taking as given the forecasts of the aggregate variables, made using the firm's information set and the laws of motion (i)
- Firm employment in (ii) evolves according to the law of motion (15) and
- The aggregate variables in (i) are consistent with the individual choices in (iii) and the worker's indifference conditions (18)-(19).

3.6 Solution under Full Information

I start by analyzing the approximate equilibrium under full information i.e. assuming all shocks are common knowledge. Note that the log-linear approximations of the equations characterizing the solution to firm's problem depend only on parameters and the relationship between aggregate variables and aggregate shocks. Importantly, they do not directly depend on variances of the idiosyncratic (or the aggregate) shock processes. Therefore, given a conjecture about aggregates, the response of agents to either aggregate or idiosyncratic shocks are unaffected by variances. Next, note that due to certainty equivalence, only the expected values of future variables are relevant to the firm's problem. Under full information, the actual realizations of shocks are assumed to be commonly known and so, expectations are common knowledge and unaffected by the variances as well. Finally, by construction, variances play no role in a linear aggregation. These features point to an equilibrium in which the magnitude of idiosyncratic shocks plays no role in aggregate dynamics. The next proposition formalizes this idea and delivers a key insight: up to a first-order approximation, fundamental heterogeneity by itself does have any implications for aggregate behavior.

Proposition 3 *Under full information, the laws of motion for aggregate variables are independent of σ_a^2 . In particular, they are identical to the case with $\sigma_a^2 = \sigma_m^2 = 0$ i.e. a representative agent economy with the same preferences and technology.*

This benchmark result stems from the fact that a log-linear approximation splits the state and optimal policies of each firm into two separate components - one arising from the history of aggregate shocks and the other from the realizations of idiosyncratic shocks. These coefficients are independent of second (or higher) moments. Upon aggregation, the component due to idiosyncratic conditions averages out to zero (because of the law of large numbers assumption). Therefore, aggregate laws of motion depend only on aggregate states and expectations of future aggregate conditions. Without informational frictions, these expectations depend only on (the commonly known) realizations of current and past aggregate productivity shocks. A representative agent version of the model using the same preferences and technology is subject to the same optimality and market clearing conditions and so generates the same relationship between aggregates and the history of aggregate shocks. With informational frictions, this logic no longer holds. Relative variances now affect the firm's forecasting problem, which changes the response of aggregate variables to aggregate shocks, which in turn affects an individual firm's optimal response to aggregate conditions - in other words, a different equilibrium emerges. In the following section, I describe the solution

algorithm to numerically solve for such an equilibrium.

4 Solution Strategy

Solving for equilibrium with dispersed information presents two challenges. The first stems from the fact that firms care not only about the realizations of the shocks but also about the responses of other firms. This linkage arises because the firm's ability to hire at any given wage depends on the wages/vacancies posted by other firms in the economy though the worker's indifference condition (18)-(19). Since this in turn depends on what other firms believe, the entire structure of higher order beliefs (what firms believe about other firms' beliefs about others actions, what firms believe about others beliefs about others beliefs and so on) becomes relevant for determining equilibrium actions. In a one-shot game (like the example in Section 2), all these higher-order beliefs are ultimately functions of a single random variable. This allowed the use of a simple method of undetermined coefficients to solve the problem. With more periods, higher-order beliefs can depend in an arbitrary way on the history of signals. As a result, the set of relevant state variables can become quite large as the number of periods increases. In particular, when past realizations are never revealed i.e. $T^* = \infty$, the need to forecast the forecasts of others leads to the well-known 'infinite regress' problem (Townsend, 1983). The evolution of the economy depends on the realizations of an infinite history of signals, making the problem intractable.

In this paper, this issue is resolved by the assumption that information is fully revealed after a finite number of periods. Now, only the history over the last T^* periods is relevant for determining the structure of higher-order beliefs. This allows me to summarize the effects of informational frictions on current equilibrium objects in the form of a solution to a finite-dimensional filtering problem. This approach for dealing with the infinite-regress follows Hellwig (2002) and Hellwig (2008a)²².

The second challenge arises because of the presence of 'physical' state variables, capital and employment. As a result, the history of signals received by a firm affects current decisions in two ways. One, since the learning problem is a dynamic one, they directly affect current forecasts of fundamentals. Two, they also determine the level of its employment and capital at the beginning of

²²Other approaches used in the literature to deal with this problem involve restricting attention to special cases where the relevant history can be summarized in a finite dimensional state variable (e.g. Woodford, 2003) or by truncating the dependence of equilibrium actions on higher order beliefs (e.g. Graham and Wright, 2010 or Nimark, 2008) or by modeling the history dependence using finite-order ARMA processes (e.g. Sargent, 1991 or Mackowiak and Wiederholt, 2010).

the period ²³. One of the contributions of this paper is to develop a method to do keep track of both these effects in a tractable manner. The methodology combines standard techniques for solving dynamic optimization problems using linear approximations with the approach for dealing with heterogeneous information from Hellwig (2002). The result is a flexible and tractable framework that can handle a rich set of individual state variables as well as complicated signal structures (whether endogenous or exogenous, public or private).

Note that all the shocks - aggregate and idiosyncratic - are assumed to be ultimately transitory, though they may be persistent to an arbitrarily high degree. Also, both capital and employment are subject to exogenous depreciation rates. Therefore, the effects of past shocks on current allocations will become arbitrarily small over time. I exploit this feature in my numerical analysis and assume that there exists some large lag T , such that shocks more than T periods old do not have any effect on current variables. In the discussion that follows, any reference to the ‘entire’ history of shocks in period t means the history of shocks up to $t - T$.

A summary of the iterative procedure that is used to solve for the approximate equilibrium is as follows:

- **Step 1:** Conjecture a (linear) relationship of aggregates to aggregate shocks
- **Step 2:** Derive full information equilibrium policy functions using a log-linear approximation
- **Step 3:** Invoke certainty equivalence to replace unknowns by their conditional expectations
- **Step 4:** Aggregate individual policy rules to express aggregate variables as (linear) functions of aggregate state variables and ‘average’ conditional expectations
- **Step 5:** Exploit normality to write ‘average’ conditional expectations as (linear) functions of aggregate shock realizations
- **Step 6:** Combine to express aggregates in terms of the aggregate shocks
- **Step 7:** Verify conjecture and iterate until convergence

Appendix B provides more details on the algorithm.

Description	Value
<i>Preferences and Production</i>	
Time period	1 month
β Discount factor	0.996
α_1 Share of capital	0.23
α_2 Share of labor	0.67
δ Depreciation of capital	0.0028
ψ Adjustment cost	30
<i>Labor Markets</i>	
δ_n Rate of exogenous destruction	0.034
η Elasticity of job filling rate	0.5
$\bar{\mu}$ Scale parameter of job filling rate	0.48
<i>Stochastic Processes</i>	
ρ Persistence of aggregate TFP	0.98
σ Standard deviation of shocks to aggregate TFP	0.005
ρ_a Persistence of idiosyncratic TFP	0.995
σ_a Standard deviation of shocks to idiosyncratic TFP	0.03
σ_m Standard deviation of shocks to efficiency	0.36

Table 1: Calibration Summary

5 Calibration

Calibration of most of the aggregate parameters is fairly standard and closely follows the strategy in Shimer (2010). The values for the discount rate β , the share of labor α_2 and the capital depreciation rate δ are borrowed directly from the real business cycle literature. The share of capital, α_1 is set to target a total share paid to factors of 90%. The adjustment cost parameter, ψ , is set to target a ratio of investment volatility to output volatility of 4.

The rate of exogenous separation δ_n is taken from Shimer (2005). Following much of the literature (e.g. see Shimer 2010), the elasticity parameter of the matching function η is set to 0.5. With the (constant) marginal cost of posting a vacancy normalized to 1, I choose the scale parameter $\bar{\mu}$ to target a monthly job-finding rate of 0.65. The main numerical results in the paper are robust to varying these parameter choices.

The persistence and standard deviation of the aggregate shock ρ correspond to the values used in the RBC literature, adjusted for the fact that a time period in this paper is a month. Next, I turn to my choice of parameters for the 2 idiosyncratic shock processes - productivity and matching. Given the AR(1) assumption, this involves picking two parameters each - the autocorrelation and the standard deviation of the innovation. The general strategy is to use cross-sectional and time-series properties of firm-level employment and vacancies to pin down these parameters²⁴. These targets are chosen conservatively - i.e. geared towards more learning and smaller effects - to the extent possible.

First, I make use of results from the large body of work documenting the empirical properties of firm growth rates in the US. Two robust findings emerge from this literature. One, there is a large cross sectional dispersion in firm level growth rates. Davis et al. (2007) report that from 1970 to 1980, the cross-sectional variance in annual growth rates ranged from 20% to 25%. I choose a conservative target of 10 % in my calibration²⁵. Second, firm growth rates are independent of firm size. This observation, commonly referred to as Gibrat's law, implies that firm-level shocks induce permanent changes in the level of the firm's employment. In the context of my model, this implies that innovations to idiosyncratic productivity have to be permanent²⁶. For my baseline calibration,

²³This channel is missing, for example, in Hellwig and Venkateswaran (2009) and Mackowiak and Wiederholt (2010).

²⁴Interpreting i , the informational unit in my model, as a firm is a natural starting point for my analysis. It seems reasonable to assume that the sources of information in my model - productivity, labor market outcomes - are most likely to be aggregated at the firm level.

²⁵The sensitivity of the numerical results to this target is very low.

²⁶This also finds support in Franco and Philippon (2007), who document large and permanent firm-specific shocks

I use a value of 0.995 for the persistence of the idiosyncratic process ρ_a . In Section 6.2, I discuss the sensitivity of the results with respect to this choice.

Next, I calibrate the idiosyncratic matching shocks. Recall that these shocks slow down learning about aggregate conditions from past labor market outcomes at the firm level. In order to focus on this informational role, I abstract from any direct effects on intertemporal incentives and assume that these shocks are completely transitory i.e.

$$m_{it} = u_{it}^m .$$

To calibrate the variance of u_{it}^m , I use recent work on vacancies and hiring by Davis, Faberman and Haltiwanger (2010). They find evidence of a strong positive relationship between a firm's employment growth rate and its vacancy yield i.e. the fraction of vacancies that are converted into new hires. The idiosyncratic matching shocks affect both firm growth and vacancy yield in the same direction (by construction) and therefore, lead to positive correlation between these quantities. Other things equal, the greater the variance of these efficiency shocks, the higher the elasticity.

Now, there could be other explanations, potentially non-random, at work behind this positive comovement²⁷. Therefore, I conservatively choose a lower elasticity target than than the observed value. In particular, I set the standard deviation of the shocks to recruiting efficiency, σ_m , to deliver an elasticity of vacancy yield to hiring of 0.5 (Davis et. al. find an elasticity of 0.72)²⁸.

A crucial parameter is the revelation lag T^* . One possible interpretation is to map this to the actual physical delay in the arrival of information about aggregates. Since aggregate data are compiled and reported very frequently (monthly for unemployment, quarterly for GDP), this leads us to rather small lags. However, this simple interpretation is a little misleading for a couple of reasons. First, reported aggregate statistics are affected by other shocks (e.g. measurement errors, other sources of uncertainty), which are not explicitly modeled in the model. Secondly, and more importantly, what matters is not the arrival of aggregate data *per se*, but the amount of information they contain about the labor market conditions facing an individual firm. In the model, *any* aggregate statistic observed without error will undo the informational friction. Obviously, this is a consequence of the highly stylized representation of aggregate uncertainty and labor markets in

along with significantly smaller, transitory aggregate shocks using data on large US firms.

²⁷For example, Davis, Faberman and Haltiwanger (2010) conjecture that growing firms might be varying an unobserved recruiting intensity per vacancy or that there are increasing returns to vacancies at the micro-level. A wage posting model with convex vacancy costs can also generate a positive correlation, but Kaas and Kircher (2011) find that this is quantitatively very small.

²⁸The numerical results are not very sensitive to this target. As an example, a calibration which delivers an elasticity as low as 0.15 delivers almost identical results.

the model. In practice, the ability of an individual firm to forecast conditions in the labor market relevant to it using highly aggregated data is likely to be very limited. The final reason draws upon the insights of the rational inattention literature. In a nominal price-setting context, Mackowiak and Wiederholt (2009) show that firms use their limited information processing capacity resources entirely to extracting information from their own sales rather than acquire direct information about aggregate shocks. The underlying reason - that aggregate shocks are an order of magnitude smaller than idiosyncratic disturbances - is present here as well. In other words, it seems reasonable to conjecture that, if firms are constrained in their ability to process information, direct learning about aggregate conditions is likely to be quite limited. Given the information structure here, this maps into a relatively long revelation lag.

Obviously, the best way out would be to explicitly model these additional aspects, but this would make the model significantly harder to solve. Therefore, I pursue a different approach in this paper and present results for a range of values for T^* . In the baseline calibration, I set T^* to 6 months and in section 6.2, I show that the quantitative results are significant even for shorter lags.

6 Results

6.1 Moments

Table 2 presents the key second moments of aggregate labor market variables in the model and compares them to their counterparts in the post-war US data. The upper panel reports the standard deviation of the annual growth rates of aggregates (relative to standard deviation of output growth). The corresponding data moments are those reported by Shimer (2010). Recall that, by Proposition 3, the response of aggregates under full information case is the same as in an economy with a representative agent. Not surprisingly then, the row labeled ‘Full Info’ confirms the findings of the literature and highlights the inability of the standard search model to generate sufficient volatility in labor market activity - the relative standard deviations of employment and market tightness are an order of magnitude lower than in the data.

Adding informational frictions improves the picture significantly. The relative volatility of (the growth rates of) both employment and market tightness are increased significantly, with the latter now in excess of observed levels. The lower panel in Table 2 repeats the exercise with log-deviations (as opposed to growth rates) and tells a very similar story - informational frictions significantly close the gap between volatilities predicted by the model and those observed in the data. Given that the model abstracts from a number of other modifications to the standard model emphasized

	Data	Full Info	Het. Info	Sticky Wages
Quarterly growth rates				
Employment	0.64	0.01	0.59	0.51
Market Tightness	14.70	0.48	29.96	21.88
Log deviations				
Employment	0.65	0.01	0.23	0.27
Market Tightness	15.30	0.75	10.75	11.20

Table 2: Standard deviation of key variables (relative to the standard deviation of output).

by other papers, this improvement in performance is quite remarkable.

Why does confusion about the nature of the shock amplify the firms' responses? The intuition is the same as in the simple example. A firm-specific productivity shock, unlike an aggregate shock, has no effect on aggregate labor market conditions. Therefore, a firm which receives a positive idiosyncratic shock expects to be more successful in its hiring efforts than an positive aggregate disturbance of similar magnitude. As a result, when it mistakes an an aggregate shock for an idiosyncratic one, it overreacts, adjusting its hiring aggressively²⁹

For purposes of comparison, Table 2 also shows the results from a model with wage stickiness. Many authors (notably Hall, 2005) have argued that the standard model generates too little cyclical movement in the incentives to create new jobs because it assumes that wages respond 'too much' to productivity shocks. In other words, wage movements - both current and expected - largely offset the changes in the value of a vacancy from the firm's perspective. If, on the other hand, wages were rigid (or sufficiently sticky), then the response of vacancies is considerably magnified. The last row of the table shows results (as reported in Shimer 2010) for a similar model. Shimer (2010) uses a random search model without heterogeneity and a wage process of the form:

$$W_t = 0.95 W_{t-1} + 0.05 W_t^{\text{Nash}},$$

where W_t^{Nash} is the wage under Nash bargaining. As the table shows, heterogeneous information leads to a greater degree of amplification in labor market volatility compared to the sticky wage model³⁰. I return to this comparison in Section 7, where I perform a more thorough comparison of the implications of the two models.

²⁹In the baseline calibration, the slightly larger persistence of firm-specific shocks relative to aggregate shocks also contributes to this amplification, but, as I will show, this effect is quantitatively small.

³⁰This is true only for the assumed degree of rigidity. As Shimer (2010) shows, if wage stickiness is sufficiently high, then that model also predicts more volatility in labor market activity than what is observed in the data.

	Full Revelation Lag, T^* , in months					
	Data	Full Info	6	3	2	1
Quarterly growth rates						
Employment	0.65	0.01	0.59	0.66	0.62	0.52
Tightness	15.30	0.75	29.96	33.56	31.72	29.28
Log deviations						
Employment	0.64	0.01	0.23	0.22	0.20	0.15
Tightness	14.70	0.48	10.75	10.49	9.94	8.50

Table 3: Standard deviation of key variables (relative to the standard deviation of output).

	Persistence of idiosyncratic productivity					
	Data	Full Info	0.995	0.98	0.95	0.5
Quarterly growth rates						
Employment	0.65	0.01	0.59	0.59	0.58	0.47
Tightness	15.30	0.75	29.96	29.66	29.16	22.49
Log deviations						
Employment	0.64	0.01	0.23	0.23	0.23	0.19
Tightness	14.70	0.48	10.75	10.80	10.70	8.32

Table 4: Standard deviation of key variables (relative to the standard deviation of output).

6.2 Robustness

In this section, I examine the sensitivity of the results to key parameter assumptions. I start with T^* , the full revelation lag. As mentioned earlier, the baseline calibration sets this parameter to 6 months. In Table 3, I report results when shocks are assumed to be revealed with much shorter lags. As the table shows, the amplification is significant even when the confusion lasts only for a very short period of time.

Next, in Table 4, I show the sensitivity of my results to the persistence of idiosyncratic productivity shocks. As in the simple example, general equilibrium linkages lead to amplification even when the persistence of the idiosyncratic shocks is less than that of aggregate shocks (recall that aggregate shocks have a persistence of 0.98). Table 4 shows that these effects are quite strong - even for relatively transitory processes, heterogeneous information significantly increases the responsiveness of employment and tightness to productivity shocks.

	y	n	θ	c	c-y
Data	1.00	0.64	14.70	0.61	0.66
Full Info	1.00	0.09	3.80	0.57	0.43
Het. Info	1.00	0.79	37.27	0.58	0.92
Sticky Wages	1.00	0.51	21.88	0.26	0.81

Table 5: Standard deviation of key variables (relative to the standard deviation of output).

7 Extension: Random Search

In this section, I analyze a variant of the above model with a number of modifications - random (instead of directed) search, labor input in the recruiting process, Nash bargaining for wages and risk aversion. In addition to demonstrating the robustness of the main amplification result, this will also allow me to compare this mechanism in this paper more directly with the sticky wage explanation. The environment closely follows the one laid out in Shimer(2010), augmented for heterogeneity and informational frictions. The prospect of additional learning from wages will turn out to be quite important. To highlight this role of wages, I study 2 cases separately. In the first version, I assume that firms and workers bargain over wages under symmetric information, ruling out the possibility that firms learn anything new from the bargaining process. In the second version, I directly make assumptions about a reduced form wage function, which gives another source of information about aggregate conditions. Except for these changes, the rest of the environment is essentially unchanged (in the interest of brevity, the detailed description of the two cases and the calibration strategy are laid out in Appendix C).

Case I (Nash bargaining): Table 5 presents the key moments. In addition to the labor market variables, the table also contains moments for consumption, the consumption-output ratio for the first case. As before, the table highlights the significant amplification potential of informational frictions. The relative volatility of employment and market tightness are an order of magnitude higher under heterogeneous information, compared to the full information benchmark.

Table 2 also presents the same moments for a very similar model with full information and a sticky wage assumption of the form:

$$W_t = 0.95 W_{t-1} + 0.05 W_t^{\text{Nash}},$$

Both models have identical outcomes in the absence of frictions. In particular, both predict relative volatilities equal to those listed under the row marked ‘Full info’. As the table shows,

heterogeneous information leads to a greater degree of amplification in labor market volatility compared to the sticky wage model³¹. However, informational frictions in this paper operate through a very different channel and therefore lead to different implications for other moments in the data. Later in the paper, I elaborate on this point using two sets of moments. In the next subsection, I show that the model with heterogeneous information leads to a negative comovement between the aggregate labor wedge and employment. This is an important finding, because generating such countercyclical movements in the labor wedge has proved to be a big challenge to the business cycle literature. The second set of moments pertains to the volatility of wages - or more precisely, the elasticity of average wages to aggregate shocks. The assumption of Nash bargaining under symmetric information dampens the response of wages to aggregate shocks. This occurs due to the fact that in equilibrium, aggregate shocks are attributed partly (in fact, largely) to idiosyncratic factors. Since firm-specific shocks have a smaller effect on wages (because, by construction, they do not affect labor market conditions - in particular, the job-finding rate and through that, the outside option of the worker), this misattribution leads to a smaller adjustment of the wage. The wage stickiness model also implies a dampened response of wages. This may be suggestive of a common mechanism, working essentially through the wage. However, this intuition is not quite correct. In the heterogeneously informed economy, the response of the economy to an aggregate shock comes largely from the responses to perceived movements in idiosyncratic factors. Therefore, the responsiveness (or lack thereof) of wages to *aggregate* shocks has only a small effect on firms' hiring decisions. I return to this point later in this section, where I show that an alternative assumption for wages can make wages more responsive to aggregate shocks and still generate amplification similar to Table 5.

Table 6 repeats the exercise with the log-deviations (as opposed to growth rates) and tells a very similar story - informational frictions significantly close the gap between volatilities predicted by the model and those observed in the data. Given that the model abstracts from a number of other modifications to the standard model emphasized by other papers, this improvement in performance is quite remarkable.

The Labor Wedge: The *aggregate labor wedge*, denoted $\hat{\tau}_t$, is defined as the deviation between the marginal product of labor and marginal rate of substitution, both computed using aggregate data:

³¹Note, however, if the degree of rigidity is sufficiently high, then the sticky wage model also overshoots on the volatility front.

	y	n	θ	c	c-y
Data	1.00	0.65	15.30	0.59	0.64
Full Info	1.00	0.07	2.80	0.76	0.33
Het. Info	1.00	0.46	18.77	0.71	0.53
Sticky Wages	1.00	0.26	11.21	0.68	0.55

Table 6: Relative Standard Deviation (Logs)

	y	n	θ	$\hat{\tau}$	c-y
Data	0.14	-0.46	-0.11	1.00	-0.49
Full Info	0.98	0.85	0.99	1.00	-0.99
Het. Info	0.04	-0.34	0.65	1.00	-0.59
Wage Rig.	0.92	0.67	0.75	1.00	-0.88

Table 7: Correlation with Labor Wedge (Growth Rates)

$$\alpha_2 \frac{Y_t}{N_t} (1 - \hat{\tau}_t) = \frac{u'(N_t)}{u'(C_t)}.$$

The wedge is thus the implicit tax on labor income when we look at the data through the eyes of a frictionless representative agent model³² Many authors have documented two key properties of this object in the post-war US data - one, it shows significant amounts of volatility and two, it comoves negatively with the cycle, in particular with aggregate employment. In other words, relative to the frictionless model, the data behave as if the implied labor tax rises significantly during recessions, exacerbating the decline in employment. The opposite happens in a boom.

The ability to match these cyclical patterns is a desirable property of a business cycle theory. The frictionless real business cycle framework with only productivity shocks implies a constant or *acyclical* wedge. Shimer (2009) surveys alternative explanations and argues that search and matching frictions are a natural framework for analyzing this wedge. However, as he shows in Shimer (2010), models with search frictions tend to induce *procyclical* movements in the wedge. This occurs because search frictions, at some level, act like labor adjustment costs and therefore, serve to dampen the response of labor to shocks.

Table 7 shows the correlation of various aggregate variables with the labor wedge $\hat{\tau}_t$. As with relative volatilities, the full information case replicates the earlier findings of the literature i.e.

³²Note that, with search frictions, the wedge $\hat{\tau}_t$ depends on the actual labor tax rate τ , but is not identically equal to it.

search frictions lead to a strongly procyclical labor wedge. Heterogeneous information acts in the opposite direction. In a boom, for example, firms attribute a positive aggregate shock to a favorable idiosyncratic disturbance and therefore, increase hiring aggressively. From the perspective of the frictionless model, they behave as if they are being subsidized, reducing the inferred tax in (7). The net effect of these two opposing forces is an aggregate labor wedge that comoves negatively with employment³³.

Table 7 also highlights an important dimension in which heterogeneous information outperforms the model with wage stickiness. The latter achieves only a modest reduction in the procyclicality of the labor wedge. For example, the correlation of the wedge with employment drops modest to 0.67 with this level of stickiness (in the data, this correlation is -0.46). Only by making wages are almost completely rigid, do we get countercyclical movements in the wedge - see Table 4.3 in Shimer (2010). Even modest delays in the arrival of information about aggregates, on the other hand, generate the right sign for this comovement.

Case II (Learning from wages): The analysis of both the benchmark model with wage-posting and the extension with symmetric Nash bargaining raises a couple of issues about the amplification mechanism. One, both the environments rule out learning from wages, potentially an important source of information about aggregate conditions. Two, in both versions, informational frictions also dampen the response of wages to aggregate shocks, relative to the respective full information benchmarks. This might give the impression that the amplification is ultimately a consequence of this ‘rigidity’ in wages³⁴. The purpose of this subsection is to address these two concerns. I modify the wage determination assumption in the model of the previous subsection. Specifically, instead of specifying a bargaining protocol (Nash) and an information structure (symmetric), I directly make assumptions about the relationship between wages and the relevant state variables. While the lack of explicit microfoundations makes this less appealing than the other two specifications in this paper, it will serve to address the two concerns referred to above. First, it will

³³Angeletos and La’o (2010a) also generate a source of countercyclical movement in the labor wedge in an economy with heterogeneous information. But, in their setup, this comes from the economy’s response to ‘noise’ in the public signal about productivity. False good news about the economy causes an increase in output and employment, without a corresponding increase in technology. This lowers the MPL and raises the MRS, leading to a drop in the observed wedge. However, in their model, informational frictions lead to *procyclical* movements in the labor wedge in response to aggregate productivity shocks - because output and employment respond less than they would under full information. Here, on the other hand, informational frictions induce a negative comovement between the labor wedge and employment.

³⁴This is true in Menzio (2005), where asymmetric information mutes the sensitivity of wages to transitory shocks and magnifies the cyclical response of vacancies and unemployment.

allow me to demonstrate the robustness of the results in the previous sections to the introduction of additional learning through wages. In order to facilitate this comparison, I adopt a form of the wage equation which, under full information, leads to the same aggregate behavior as the case with Nash bargaining under symmetric information. Under heterogeneous information, however, the two specifications will have very different implications, arising primarily due to differences in the extent of learning. Second, this exercise will help distinguish the mechanism at work in this paper from the sticky wage hypothesis. In particular, under the wage specification I adopt, the average wage rate in the economy with heterogeneous information will be *more* responsive to aggregate shocks than in the full information benchmark, revealing that the amplification generated by heterogeneity in information is not coming from a dampened response of wages to aggregate shocks. The wage equation is given by

$$(1 - \tau)W_{it} = \phi(1 - \tau)\frac{\alpha_2 Y_{it}}{N_{it}(1 - V_{it})}(1 + \Theta_{it}) + (1 - \phi)Z_{it}C_t\gamma. \quad (23)$$

The expression is essentially a modified-version of the representative agent wage equation (the details are in the Appendix) and has the convenient property that the approximate equilibrium converges to the Nash bargaining case as informational frictions disappear. While this reduced form approach lacks the appeal of the microfounded wage protocols of the previous sections, it offers a clean way to bring out the role of learning from wages and the elasticity of wages to aggregate shocks.

	y	n	θ	c	c-y
Data	1.00	0.64	14.70	0.61	0.66
Full Info	1.00	0.09	3.66	0.58	0.42
Sym. Bargaining	1.00	0.79	37.27	0.58	0.92
Wage Equation	1.00	0.36	18.24	0.46	0.84
Wage Equation *	1.00	0.76	35.52	0.54	0.95

Table 8: Standard deviation of growth rates, relative to that of output

* : Without learning from wages

Table 8 compares the model-predicted relative standard deviations for this version. It shows that amplification is still present, but is more subdued under the reduced form wage specification. This difference is almost entirely from the additional information that wages now convey (a role that was ruled out by assumption with symmetric bargaining). To show this more clearly, the last row presents results using the same wage function, but turning off learning from wages. Now, with

this channel turned off, the reduced form wage equation and symmetric bargaining assumption yield very similar results.

This last experiment also illustrates an important distinction between heterogeneous information and wage stickiness as means to amplify labor market volatility. The third and last rows in Table 8 imply very different responses of average wages to aggregate shocks. However, as the table shows, they lead to almost identical implications for labor market volatility, showing that a dampened wage elasticity is not central to the amplification generated by informational frictions. This is an important finding, particularly in light of the difficulties associated with empirically inferring the extent of stickiness in wages. For example, Pissarides (2009) and Haefke et. al. (2012) find little evidence in the micro data for stickiness of wages paid to newly hired workers, which is the relevant measure of rigidity from the perspective of job creation.

8 Conclusion

The analysis in the preceding sections, while tightly focused on a search and matching context, relies on a more general mechanism - when there are strong equilibrium linkages, imperfect information about the nature of aggregate disturbances can have a significant effect on outcomes. Information structures based on market signals offer a natural and intuitive example of this type of friction.

There are several interesting avenues for future research. One direction is towards a more complete picture of the effect of informational frictions on the labor market decisions. This paper focuses exclusively on the job-creation margin and so, abstracts from a number of other aspects of labor market activity, such as endogenous job destruction and on-the-job search. An investigation of the interactions between these channels and informational frictions holds a lot of promise, for understanding both aggregate behavior and the cross-sectional distribution. Another important direction is to investigate the implications of market-based information structures to other choices (e.g. investment³⁵, portfolio choice and asset pricing).

The tight connection between informational and fundamental heterogeneity in this paper also suggests strategies for empirical validation of the mechanism. For example, as mentioned earlier, the model predicts that countries where firms are subject to larger shocks are also more likely to exhibit a greater sensitivity of hiring to aggregate shocks. A similar implication can be derived with respect to labor market integration, i.e. more integrated labor markets are likely to display greater aggregate volatility. Testing these implications using data across countries or sectors is a

³⁵See Graham and Wright (2010).

promising agenda.

On the informational front, allowing agents to endogenously choose the extent of learning about various shocks is a natural next step. This would provide support for a key assumption in this paper - learning, whether about aggregate or idiosyncratic shocks, occurs predominantly through market signals. To see this, suppose firms had a limited amount of resources (e.g. cognitive ability in the spirit of the rational inattention literature) to devote to gathering information, but are free to allocate those resources to various sources of information (e.g. market signals versus other, potentially more direct, sources). In such an environment, it seems reasonable to conjecture that firms devote most of their attention to signals that are closely tied to their own activities. In other words, since information from market activities contain a lot of information about idiosyncratic shocks that are both large and payoff-relevant, firms will find it optimal to track them as closely as possible. Direct sources of information about aggregate shocks, on the other hand, will receive very little attention because their relative size makes them relatively less important to profitability³⁶.

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³⁶Mackowiak and Wiederholt (2009) arrive at a similar conclusion in a nominal price-setting application. They show that firms find it optimal to pay no attention to direct signals about aggregate nominal shocks and acquire all their information from their sales (which contains information about both aggregate and idiosyncratic shocks).

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Appendix A Equilibrium Definition

An equilibrium is a set of

- i. Aggregate allocations $N_t, V_t, S_t, \tilde{S}_t$
- ii. Firm-level employment N_{it}
- iii. Firm decisions $K_{it+1}, W_{it}, V_{it}, D_{it}$

such that

- The choices in (iii) solve the firm's problem, taking as given the forecasts of the aggregate variables, made using the firm's information set and the laws of motion (i)
- Firm employment in (ii) evolves according to the law of motion (15) and
- The aggregate variables in (i) are consistent with the individual choices in (iii) and the worker's indifference conditions (18)-(19).

Appendix B Solution Algorithm

Define

$$\Omega_{it} \equiv \begin{pmatrix} U_t \\ U_{it}^a \\ U_{it}^m \end{pmatrix} \text{ where } U_t \equiv (u_t, u_{t-1}, \dots, u_{t-T})' .$$

and U_{it}^j , $j = a, m$ are defined analogously.

Step 1: The effect of aggregates on an individual firm's problem can be summarized by the market utility of the worker, S_t . The starting point of the algorithm is a conjecture for the (linear) relationship between this aggregate variable and the history of aggregate shocks:

$$s_t \equiv \log S_t - \log \bar{S} = P U_t , \tag{24}$$

where P is a $1 \times T$ matrix. Note that this specification allows S_t , up to a log-linear approximation, to depend on the history of aggregate shocks in an arbitrary way.

Step 2: Next, I solve the problem of firm i , assuming it is perfectly informed i.e. assuming it knows the entire history of aggregate and idiosyncratic shocks affecting it. The firm enters period t with its current capital stock k_{it} as well as the level of employment and choice of recruiting effort

in $t - 1$, n_{it-1} and v_{it-1} respectively. The last two are state variables because they affect the level of employment in the current period. In addition, the entire history of innovations to all the shock processes affecting the firm's payoffs (Ω_{it}) will affect the firm's decisions. These shocks are relevant not only because they directly affect the firm's current productivity, wages etc., but also because they form the basis for the firm's forecasts of future values of aggregate and firm-specific factors. The solution to this problem can be expressed in the form of a law of motion for the firm's state and policy variables:

$$X_{it} = BX_{i,t-1} + D\Omega_{it} \quad \text{where } X_{it} \equiv \begin{pmatrix} k_{it+1} \\ n_{it} \\ v_{it} \\ w_{it} \\ d_{it} \end{pmatrix}. \quad (25)$$

Step 3: The next step makes use of certainty equivalence implicit in the linear approximation of the policy rules and replaces the actual realizations in (25) with conditional expectations.

$$X_{it} = BX_{i,t-1} + D\mathbb{E}_{it}\Omega_{it}. \quad (26)$$

Step 4: To derive the laws of motion for the aggregate state variables, I add (26) over all i . The result is a law of motion which depends only on aggregate state variables and 'average' expectations about aggregate and firm-specific shocks in the economy.

$$X_t = BX_{t-1} + D\bar{\mathbb{E}}_t\Omega_{it} \quad \text{where } \bar{\mathbb{E}}_t(\cdot) = \int \mathbb{E}_{it}(\cdot) di. \quad (27)$$

Step 5: Next, I characterize the average expectations in this economy. To achieve this, note that the signals received firm i are linear combinations of Ω_{it} i.e., for a suitably defined Γ ,

$$\varpi_{it} = \Gamma \Omega_{it}.$$

Then, standard filtering results using normally distributed variables imply that

$$\mathbb{E}_{it}\Omega_{it} \equiv \mathbb{E}(\Omega_{it}|\varpi_{it}) = \Sigma \Gamma' (\Gamma \Sigma \Gamma')^{-1} \varpi_{it} \quad (28)$$

$$= \Sigma \Gamma' (\Gamma \Sigma \Gamma')^{-1} \Gamma \Omega_{it}, \quad (29)$$

where Σ is the variance-covariance matrix of Ω_{it} . Average expectations are given by

$$\begin{aligned} \bar{\mathbb{E}}_t(\Omega_{it}) &= \int \mathbb{E}_{it}\Omega_{it} di = \Sigma \Gamma' (\Gamma \Sigma \Gamma')^{-1} \Gamma \int \Omega_{it} di \\ &= \Sigma \Gamma' (\Gamma \Sigma \Gamma')^{-1} \Gamma \bar{\Omega}_t, \end{aligned}$$

where $\bar{\Omega}_{it}$ is the cross-sectional average of the shocks. Since we adopt the usual law of large numbers convention for the idiosyncratic shock realizations,

$$\bar{\Omega}_t = \begin{pmatrix} U_t \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} .$$

Average expectations thus can be written as a function only of the aggregate shock vector U_t i.e.

$$\bar{\mathbb{E}}_t(\Omega_{it}) = H U_t .$$

Step 6: Substituting the expression for average expectations back into (27),

$$X_t = B X_{t-1} + D H U_t . \quad (30)$$

Step 7: Equation (30) implies a relationship between the market utility S_t and the realizations of the aggregate shock U_t :

$$S_t = \tilde{P} U_t . \quad (31)$$

If $\tilde{P} = P$, the initial guess, then the algorithm has converged and the allocations implied by (26), (29) and (30) constitute an approximate equilibrium as defined in Section 3.5. If not, i.e. $\tilde{P} \neq P$, then we repeat the above steps using a new conjecture:

$$P_1 = g P + (1 - g) \tilde{P} , \quad g \in (0, 1) .$$

Appendix C A Model of Random Search

Households: There is a representative household with a measure 1 of members, who work in one of a continuum of markets i . The household maximizes the expected discounted sum of utilities of all its members

$$\sum_t \int_{s^t} \beta^t \Pi(s^t) \left(\ln C_t(s^t) - \gamma \int N_{it}(s^t) di \right) ,$$

where $C_t(s^t)$ is aggregate household consumption, $N_{it}(s^t)$ is employment in i . The measure of employed members of the household, $N_t(s^t)$, is :

$$N_t(s^t) \equiv \int N_{it}(s^t) di .$$

The household is assumed to have access to complete contingent claims markets and faces a lifetime budget constraint:

$$\sum_t \int_{s^t} Q_t(s^t) C_t(s^t) = \sum_t \int_{s^t} Q_t(s^t) \int (W_{it}(s^t) N_{it}(s^t) + D_{it}(s^t)) di .$$

where $Q(s^t)$ is price of an Arrow-Debreu security which pays off 1 unit in state s^t , W_{it} and D_{it} denote the wage rate and dividends paid out by the representative firm in i . Standard optimization arguments imply

$$\lambda Q_t(s^t) = \frac{1}{C_t(s^t)} ,$$

where λ is the multiplier on the lifetime budget constraint.

Firms: As before, firms maximize expected discounted dividends. Apart from the wage determination protocol, there are 2 differences in the firm's problem compared to the benchmark model. The first is that these profits are now weighted by the (expected) stochastic discount factor of the household. Note that the firm does not observe the discount factor directly - it forms expectations about the household's marginal utility of consumption, based on its forecast of the aggregate conditions. The second difference is that recruiting is now modeled as a time-intensive activity. The production function of the firm is still Cobb-Douglas, with output given by:

$$Y_{it} = A_t A_{it} K_{it}^{\alpha_1} (N_{it}(1 - V_{it}))^{\alpha_2}$$

where V_{it} is the fraction of the firm's labor force allocated to recruiting efforts. Recall that, earlier, the cost of recruiting was denominated in terms of output. Here, I follow Shimer (2010) and assume that the firm devotes a fraction of its labor force to recruiting.

The law of motion for employment in i is given by:

$$N_{i,t} = (1 - \delta_n) N_{i,t-1} + V_{i,t-1} N_{i,t-1} F_{i,t-1} . \quad (32)$$

where δ_n is the (exogenous) separation rate and $F_{i,t-1}$ is the (endogenous) number of hires each recruiter deployed in i in period $t-1$ is able to attract. The firm takes as given $F_{i,t-1}$ while choosing $V_{i,t-1}$. The household, on the other hand, takes this law of motion as completely exogenous.

The effectiveness of each recruiter in the economy depends on aggregate labor market conditions, but is also subject to an idiosyncratic shock, similar to one used in the benchmark model.

$$F_{it} = \bar{\mu} (\Theta_t)^{-\eta} M_{it}, \quad \eta \in (0, 1) \quad (33)$$

$$(34)$$

where, $\Theta_t \equiv \frac{V_t N_t}{1 - N_t}$ measures market tightness. The expected efficiency of a firm's recruiting efforts is clearly declining in Θ_t , the key equilibrium interaction in this version of the mode.

Wages: The standard assumption for wage determination in the random search literature is a Nash bargaining protocol. Applying that to the environment of this paper presents a number of challenges. The first is the need to specify - and impose discipline on - what firms and workers know about fundamentals as well as each others' information. Unlike the natural assumption that firms use the information generated by their market activities to form forecasts, there is no obvious way to model the interconnections between information sets of firms and the workers they hire. The second issue is one of tractability. Existing work on bargaining under informational frictions (Brugemann and Moscarini 2010, Kennan 2010) use environments that are much simpler and more stylized than the one studied in this paper.

For these reasons, I take a different approach. I assume that wages are determined through period-by-period Nash bargaining assuming that workers and firms are symmetrically informed³⁷. This avoids the tractability problems caused by informational asymmetries, but also implies that wages contain no additional information³⁸.

Under symmetric (but imperfect) information, it is possible to show that the Nash bargaining leads to the following expression for the after-tax wage:

$$(1 - \tau)W_{it} = (1 - \phi) \frac{\mathbb{E}_{it}(\gamma Z_{it} + \Upsilon_t)}{\mathbb{E}_{it}Q_t} + \phi \frac{\alpha_2 Y_{it}(1 - \tau)}{N_{it}(1 - V_{it})}, \quad (35)$$

where Υ_t is the future value to the household of an unemployed worker and ϕ indexes the bargaining power of the worker. Thus, the wage is a weighted average of the expected value of an additional worker to the firm and the household, with the weights determined by the bargaining power parameter, ϕ .

Information Structure: As in the benchmark model, I assume that all shocks become common knowledge after a lag of T^* periods. Apart from these, firms only have access to variables which arise in the natural course of their business - productivities and outcomes of their labor market activities. Formally, I assume that firm i 's information set at time t is given by

- Productivities: $\{a_{t-\tau} + a_{i,t-\tau}\}_{\tau=0}^{\infty}$

³⁷One way to interpret this is that each firm bargains with an in-house agent of the representative household. The agent does not communicate with the household and has access to only the information that the firm has. In this interpretation, workers who move across firms (or between the the unemployment pool and firms) from period to period are assumed to carry no information with them.

³⁸Later in this section I relax this assumption.

- Wages: $\{W_{i,t-\tau}\}_{\tau=0}^{\infty}$
- All firm-specific variables: $\{V_{i,t-\tau-1}, K_{i,t-\tau}, N_{i,t-\tau}\}_{\tau=0}^{\infty}$
- $\{U_{t-T^*-\tau}, U_{it-T^*-\tau}, Z_{it-T^*-\tau}, L_{it-T^*-\tau}\}_{\tau=1}^{\infty}$

Again, as before, I study an approximate equilibrium in the neighborhood of the deterministic steady state. Under the above assumptions, Proposition 2 extends naturally to this environment - the full information equilibrium is the same as in a representative agent economy with random search and wages determined through Nash bargaining. I employ the solution strategy outlined in Section 4 to solve for the equilibrium in the dispersed information case.

Calibration: The strategy for picking preference and technology parameters is based largely on Shimer (2010). The values for the discount rate β , the share of labor α_2 and the capital depreciation rate δ are borrowed directly from the real business cycle literature. The share of capital, α_1 is set to target a total share paid to factors of 90%. The persistence of the aggregate shock ρ also corresponds to the values used in the RBC literature, adjusted for the fact that a time period in this paper is a month.

The rate of exogenous separation δ_n is taken from Shimer (2005). Hagedorn and Manovskii (2008) estimate that hiring a worker costs about 4% of a worker's quarterly wage, which implies that each recruiter attracts 25 workers on average per quarter (i.e. the monthly counterpart $\bar{F} = 8.33$). The following steady-state relationship then pins down the fraction of workers engaged in recruiting in the steady state:

$$\delta_n \bar{N} = \bar{V} \bar{N} \bar{F} \quad \Rightarrow \quad \bar{V} = \frac{\delta_n}{\bar{F}}.$$

Given a target for steady state employment ($\bar{N} = 0.95$), the estimate for \bar{V} determines the steady state tightness, $\bar{\Theta}$. The matching function in this paper takes as input recruiting effort, instead of the usual measure of vacancies posted, so there are no direct estimates of the elasticity available. I follow Shimer (2010) and consider the symmetric case i.e. $\eta = 0.5$. Given this choice, equation (33) pins down the scale parameter $\bar{\mu}$. The worker's 'share' of the surplus ϕ is set to 0.5. The disutility of leisure, γ , is chosen so that steady state unemployment is 5%. Finally, the labor tax τ is set to match the average marginal tax rate. The values for the parameters are collected in Table 9. For the idiosyncratic shocks to productivity and recruiter efficiency, I target the same moments from the cross-sectional distribution of firm growth and vacancy yields discussed in Section 5.

Description	Value
<i>Preferences and Production</i>	
Time period	1 month
β Discount factor	0.996
α_1 Share of capital	0.23
α_2 Share of labor	0.67
δ Dep. of capital	0.0028
γ Disutility of leisure	0.43
ρ Persistence of agg. TFP	0.98
<i>Labor Markets</i>	
δ_n Rate of exogenous destruction	0.034
τ Labor tax	0.4
ϕ Workers' share of surplus	0.5
η Elasticity of job filling rate	0.5
$\bar{\mu}$ Scale parameter of job filling rate	2.32

Table 9: Calibrated values of aggregate parameters

C.1 Learning from wages

In this version, I modify the wage determination assumption in the model of the previous subsection and directly make assumptions about the relationship of wages to aggregate and idiosyncratic variables. My starting point is a representative-agent version of the model in the previous subsection. For sufficiently small aggregate shocks, Nash bargaining leads to the following expression for wages:

$$(1 - \tau)W_t = \phi(1 - \tau) \frac{\alpha_2 Y_t}{N_t(1 - V_t)} (1 + \Theta_t) + (1 - \phi)C_t \gamma, \quad (36)$$

where, as before, ϕ is a parameter indexing the bargaining power of the worker and τ is the constant labor tax rate. This expression has an intuitive interpretation - the wage is then a weighted average of the marginal product of labor (adjusted for the dynamic nature of the hiring decision) and the marginal rate of substitution between consumption and leisure, with the relative weights determined by ϕ .

Under heterogeneity, I assume that the wage determination equation takes the same form as (36), with some modifications:

$$(1 - \tau)W_{it} = \phi(1 - \tau) \frac{\alpha_2 Y_{it}}{N_{it}(1 - V_{it})} (1 + \Theta_{it}) + (1 - \phi)Z_{it}C_t \gamma. \quad (37)$$

where Z_{it} is an idiosyncratic shock. Note that, with heterogeneity, we can no longer microfound this as the outcome of a bargaining protocol. Nevertheless, under full information, the log-linear approximation of the average wage rate will be the same as in the representative agent case. This feature ensures that, under full information, the approximate equilibrium coincides with that of the previous model.

Equation (37) has another important implication. Observing (the log-linear approximation of) the wage is informationally equivalent to observing a linear combination of aggregate consumption C_t , Θ_t and the idiosyncratic noise Z_{it} . Unlike the earlier models in the paper, wages now contain additional information about aggregate conditions, but this information is partly clouded by the idiosyncratic shock.

To discipline the size of the idiosyncratic shock Z_{it} , I make use of its implications for the distribution of wages³⁹. From the wage specification (37), it is easy to see that z_{it} is a source of variation in wages, orthogonal to both the firm-specific and aggregate components of the wage. A number of papers have documented a significant role for such a component in explaining the cross-sectional distribution of wages. Abowd, Kramarz and Margolis (1999) interpret this as a person-specific effect and find that it accounts for as much as 50-60% of wage dispersion in the French data. Postel-Vinay and Robin (2002) estimate a structural model using French labor market data and find that person-effects can account for as much as 40% of wage dispersion at high skill levels though this share declines quite sharply for low skilled jobs. Davis et al. (1991) report that within-plant heterogeneity explains as much as 35-40% of the variance in wages.

Obviously, to assume that this heterogeneity is entirely unobservable is not reasonable. For example, under the person-effect interpretation, one could argue that firms might have access to certain characteristics (unobservable to the econometrician) of their workers and so may have some information about the realization of z_{it} . In the limit, if they could observe the sources of heterogeneity perfectly, wages will fully reveal the aggregate state, undoing the informational frictions entirely. In light of this, I take a conservative approach and calibrate the relative variance of z_{it} so that it accounts for only 25% of the cross-sectional variance in wages.

³⁹Nimark (2008) also makes use of a similar interpretation to find empirical counterparts for idiosyncratic marginal cost shocks.

Appendix D Proofs of Results

D.1 Proof of Propositions 1 and 2

Follows from a direct comparison of the expressions for ϕ^{Full} and ϕ^{Het} .

D.2 Proof of Proposition 3

Under full information, the log-linearized optimality and equilibrium conditions can be represented as the solution to a dynamic system of the form

$$R_1 X_{it} = R_2 X_{it-1} + R_3 \Omega_{it}$$

where X_{it} is a vector of all state and endogenous variables of interest, both aggregate and firm-specific. Integrating over all i yields

$$R_1 X_t = R_2 X_{t-1} + R_3 \bar{\Omega}_t \tag{38}$$

where $\bar{\Omega}_t$ is the cross sectional average of shocks and, by the law of large numbers assumption, is simply $[U_t \ \mathbf{0} \ \mathbf{0}]'$. Subtracting one equation from the other,

$$R_1 [X_{it} - X_t] = R_2 [X_{it-1} - X_{t-1}] + R_3 [\bar{\Omega}_t - \Omega_{it}] \tag{39}$$

Now, a solution to the original dynamic system can be found by simply adding the solutions to the two decoupled systems (38) and (39) separately. To show that they are indeed decoupled, I simply note that the driving error processes for the two systems are different. Equation (38) is driven solely by the aggregate shocks while (39) is affected only by realizations of the idiosyncratic shocks.