Setting the Right Prices for the Wrong Reasons

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June 2009
Introduction

- Incomplete information as a source of monetary non-neutrality
  - Phelps (1970) and Lucas (1972)

- Heterogeneous information + Complementarity in pricing $\rightarrow$ amplification and persistence of real effects of monetary shocks
  - Role of higher-order expectations - Woodford (2001), Mankiw and Reis (2002)
  - Rational inattention - Sims (2003), Mackowiak and Wiederholt (2008)
This Paper

- A quantitative evaluation of monetary models with heterogeneous information
- Both aggregate and idiosyncratic shocks with market-generated signals
- A model calibrated to price and quantity moments unable to generate meaningful real effects
  - Firms attribute aggregate shocks to persistent idiosyncratic factors and respond quickly
- Money is almost neutral even with severe restrictions on information
Related Literature

- Monetary models with heterogeneous information
  - Woodford (2001), Mackowiak and Wiederholt (2008)

- Calibration
  - Target moments from Burstein and Hellwig (2007), Midrigan (2007), Eichenbaum et al. (2008)

- Solution Method
  - Formulating and solving GE models with heterogeneous information
Outline

- A simple static example
- The full model
- Calibration and quantitative results
- Extensions: Productivity shocks
A Simple Example

Optimal pricing decision of firm $i$

\[ p_i = \mathbb{E}_i (p_i^*) + r \mathbb{E}_i (p - p^*) \]

where

- $p_i^*$ is the exogenous stochastic targets for firm $i$'s price
- $p = \int p_i \, di$ is the average price level
- $p^*$ is the target for the average price level
- $r \in (-1, 1)$ is the degree of pricing complementarity
Example (contd.): Targets

- The firm’s target price

\[ p_i^* = \gamma z_i + m, \text{ and hence } p^* = m \]

where
\[ m \sim \mathcal{N}(0, \sigma_m^2) \text{ and } z_i \sim \mathcal{N}(0, \sigma_z^2) \] are aggregate and idiosyncratic shocks respectively.
\[ \gamma \] measures the firm’s desired response to the idiosyncratic shock.

- The firm’s optimal rule can be written as

\[ p_i = \gamma \mathbb{E}_i[z_i] + \mathbb{E}_i[m] + r \mathbb{E}_i[p - m] \]
\[ = \gamma \mathbb{E}_i[z_i] + (1 - r) \mathbb{E}_i[m] + r \mathbb{E}_i[p] \]
Example (contd.): Signal and Equilibrium

- Firms observe a noisy signal

\[ s_i = z_i + m + \zeta_i \]

where \( \zeta_i \sim \mathcal{N}(0, \sigma^2_\zeta) \) represents idiosyncratic signal noise

- Equilibrium
  - Conjecture a linear pricing rule \( p = km \)
  - Verify the conjecture and solve for \( k \)
Example (contd.): $\gamma = 0$

$$k = \frac{(1 - r) \sigma_m^2}{(1 - r) \sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2}$$

Proposition

When $\gamma = 0$, a higher degree of complementarity ($\frac{\partial k}{\partial r} < 0$), or a lower precision of the idiosyncratic shock reduce the response of prices to aggregate shocks.

- $r = 0$: Prices adjust according to the information content in the signal i.e. the signal-to-noise ratio
- $r > 0$: Information about the aggregate shock in the signal is discounted to reflect the complementarity in pricing
- $r \to 1$: Discount becomes arbitrarily large i.e. firms increasingly disregard private signals because other firms do
Example (contd.): $\gamma > 0$

\[
k = \frac{(1-r)\sigma_m^2}{(1-r)(\sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2)} + \gamma \frac{\sigma_z^2}{(1-r)(\sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2)}
\]

The ‘right’ reasons

\[
= k_W + (1 - k_W)R \quad \text{where} \quad k_W \equiv \frac{(1-r)\sigma_m^2}{(1-r)(\sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2)}
\]

The ‘wrong’ reasons

**Proposition**

*An increase in $\gamma$ increases the response of prices to aggregate shocks ($\frac{\partial k}{\partial \gamma} > 0$). The response is bounded below by $R \equiv \frac{\gamma\sigma_z^2}{\sigma_z^2 + \sigma_\zeta^2}$.***

- $R < 1 \Rightarrow$ Complementarities still delay adjustment, but an increase in $\gamma$ mitigates their effects.
- $R = 1 \Rightarrow$ Full adjustment i.e. $k = 1$. Firms set the right prices, albeit for the
Example (contd.): Price Dispersion

\[
\text{Dispersion} = \left\{ \int (p_i - \bar{p})^2 \, di \right\}^{1/2} = k \sqrt{\sigma_z^2 + \sigma_\zeta^2}
\]

\[
= \left[ \frac{(1 - r) \sigma_m^2 + \gamma \sigma_z^2}{(1 - r) \sigma_m^2 + \sigma_z^2 + \sigma_\zeta^2} \right] \sqrt{\sigma_z^2 + \sigma_\zeta^2}
\]

- \( \gamma = 0 \quad \Rightarrow \quad \text{Dispersion} \leq \frac{\sqrt{1-r}}{2} \sigma_m < \sigma_m \)
- \( \gamma \neq 0 \)

High \( \gamma \quad \Rightarrow \quad \text{High dispersion, High } k \)

Low \( \gamma \), High \( \sigma_z^2 \quad \Rightarrow \quad \text{High dispersion, Low } k \)

For large real effects AND price dispersion, we need low \( \gamma \) and high \( \sigma_z \)
Example (contd.): Implications of Non-Separability

Suppose firms learnt about aggregate and idiosyncratic shocks separately

\[ s_1^i = m + u_i \quad s_2^i = z_i + \zeta_i \]

Equilibrium pricing rule is \( p_i = ks_1^i + ls_2^i \) where

\[ k = \frac{(1 - r)\sigma_m^2}{(1 - r)\sigma_m^2 + \sigma_u^2} \quad l = \frac{\gamma \sigma_z^2}{\sigma_z^2 + \sigma_\zeta^2} \]

Price dispersion \( Disp = k\sigma_u + l\sqrt{\sigma_z^2 + \sigma_\zeta^2} \)

High \( \sigma_u \) \( \Rightarrow \) low \( k \) and High \( \sigma_z \) \( \Rightarrow \) high price dispersion
The Full Model

- A natural interpretation to the information structure
- Richer set of micro moments to inform the choice of parameters
- Both aggregate and firm-specific shocks
- Gradual learning about shocks over time
- Quantitative Analysis
Households

The representative household maximizes the objective function

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\psi}}{1 - \psi} + \ln \frac{M_t}{P_t} - \frac{\eta}{1 + \kappa} \int_0^1 Z_{it} N_{it}^{1+\kappa} \, di \right),$$

s.t. a lifetime budget constraint

$$M_0 \geq \mathbb{E}_0 \sum_t \lambda_t \{ C_t P_t + i_t M_t - \int W_{it} N_{it} \, di - \Pi_t - T_t \}$$

where

$N_{it}$, $Z_{it}$ and $W_{it}$ denote labor supply, an idiosyncratic preference shock and nominal wage rate for labor of type $i$.

$\lambda_t$ is the stochastic discount factor.
Monetary Process

We assume that the log of money supply follows a random walk i.e.

\[ \ln M_{t+1} = \ln M_t + \mu + \sigma_u u_t \]

⇒ Constant nominal interest rates ⇒ The FOC of the household simplify to

\[ \lambda_t = \beta_t \frac{1}{M_t} \]

\[ C_t = K_0 \left( \frac{M_t}{P_t} \right)^{\frac{1}{\psi}} \]

\[ W_{it} = K_1 M_t Z_{it} N_{it}^{\kappa} \]

where \( K_0 \) and \( K_1 \) are constants that do not depend on time.
Production

The single final good is a composite of a continuum of intermediate goods

\[ C_t = \left( \int B_{it}^\frac{1}{\theta} C_{it}^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}} \]

where \( B_{it} \) is an idiosyncratic shock process. Demand functions for sector \( i \) and the aggregate price index are

\[ C_{it} = B_{it} C_t \left( \frac{P_{it}}{P_t} \right)^{-\theta} \quad P_t = \left( \int B_{it} P_{it}^{1-\theta} \, di \right)^{\frac{1}{1-\theta}} \]

Intermediate production is linear in the labor input i.e.

\[ Y_{it} = A_{it} N_{it} \]

For now, we assume \( A_{it} = 1 \).
Price Setting

The intermediate producer solves

$$\max_{P_{it}} \mathbb{E}_{it} \left[ \lambda_t \beta^{-t} (P_{it} C_{it} - N_{it} W_{it}) \right]$$

The log of the FOC (after substitution from the household optimality conditions)

$$p_{it} = \text{Const} + \frac{1}{1 + \theta \kappa} \mathbb{E}_{it} [\kappa b_{it} + z_{it}] + \mathbb{E}_{it} [m_t] + \left( \frac{\kappa (\theta - \psi^{-1})}{1 + \theta \kappa} \right) \mathbb{E}_{it} [p_t - m_t]$$

Recall the optimal pricing rule from the simple example

$$p_i = \gamma \mathbb{E}_i [z_i] + \mathbb{E}_i [m] + r \mathbb{E}_i (p - m)$$
Stochastic Processes for Idiosyncratic Shocks

We model both the idiosyncratic shocks as consisting of an AR(1) component and a transitory noise term.

\[ b_{it} = \sigma_b \sum_{s=0}^{\infty} \rho_b^s v_{i,t-s}^1 + \tilde{\sigma}_b \tilde{v}_t^1 \]

\[ z_{it} = \sigma_z \sum_{s=0}^{\infty} \rho_z^s v_{i,t-s}^2 + \tilde{\sigma}_z \tilde{v}_t^2 \]

where \( v_t^1, \tilde{v}_t^1, v_t^2, \) and \( \tilde{v}_t^2 \) are iid random variables distributed according to \( \mathcal{N}(0,1) \)
Information Structure

- Firm $i$ only observes its own demand $C_{it}$ and wage bill $W_{it}N_{it}$
- Using the household FOC and the demand function, the signals (in logs) are
  \[ x_{it} = b_{it} + \psi^{-1}m_{t} + (\theta - \psi^{-1})p_{t} \]
  \[ \omega_{it} = m_{t} + z_{it} \]
- Each period, the firm sets prices **before** markets open and the current realizations of the wage and demand signals are revealed
- Prices are set based on conditional expectations of previous aggregate shocks and persistent components of idiosyncratic shocks
Information Structure: Infinite Regress Problem

- Higher order expectations matter for optimal price decision and correct interpretation of signals

- No finite-dimensional recursive structure

- We propose a simple and computationally efficient algorithm, following Hellwig (2002) and Hellwig (2008a)
  - Key assumption: all shocks become common knowledge after T periods
  - Information set when setting $P_{it}$

$$\mathcal{I}_t^i = \{ x_{it-s}, \omega_{it-s}, u_{it-T-s}, v^1_{i,t-T-s}, \tilde{v}^1_{i-T-s}, v^2_{i,t-T-s}, \tilde{v}^2_{i-T-s} \}_{s=1}^{\infty}.$$
Solution Algorithm: Vector Notation

The ‘relevant’ shocks

\[ U_t' = (u_{t-1}, u_{t-2}, \ldots u_{t-T}) \]
\[ V_{it-1}' = (v_{it-1}^1, v_{it-2}^1, \ldots v_{it-T}^1) \]
\[ V_{it-1}' = (v_{it-1}^2, v_{it-2}^2, \ldots v_{it-T}^2) \]

and the ‘relevant’ signals

\[ X_{it}' = (x_{i,t-1}, x_{i,t-2}, \ldots x_{t-T}) \]
\[ \Omega_{it}' = (\omega_{i,t-1}, \omega_{i,t-2}, \ldots \omega_{t-T}) \]
Solution Algorithm: Common Knowledge

- \( \hat{p}_{it} \equiv \) The common knowledge component of the optimal price, captures the effect of all the fully revealed shocks
- The firm's optimal price can be split into two parts

\[
p_{it} = \hat{p}_{it} + \sigma_u (1 - r) 1' E_{it}[U_t] + r (E_{it}[p_t] - \hat{p}_t)
\]

\[
+ \sigma_b \frac{\kappa \rho_b}{1 + \theta \kappa} \gamma_b' \ E_{it}[V_{it}^1] + \sigma_z \frac{\rho_z}{1 + \theta \kappa} \gamma_z' \ E_{it}[V_{it}^2]
\]

where
\[
\gamma_b' \equiv (1, \rho_b, \rho_b^2, ..., \rho_b^{T-1}) \text{ and } \gamma_z' \equiv (1, \rho_z, \rho_z^2, ..., \rho_z^{T-1})
\]
- Vectors \( \gamma_b \) and \( \gamma_z \) are analogous to \( \gamma \) in the simple example
Solution Algorithm: Price Conjecture

We conjecture that aggregate prices can be written in the following form

\[ p_t = \hat{p}_t + \sigma_u \phi' U_t \]

for some \( T \times 1 \) vector \( \phi' = (\phi_1, \ldots, \phi_T) \).

Substituting in the optimal price condition and averaging across firms,

\[ \sigma_u \phi' U_t = \sigma_u [(1 - r) 1' + r \phi'] \bar{E}_t[U_t] + \sigma_b \gamma_b' \bar{E}_t[V_{it}^1] + \sigma_z \gamma_z' \bar{E}_t[V_{it}^2] \]

where \( \bar{E}_t[\cdot] = \int E_{it}[\cdot]di \).
Solution Algorithm: Average Expectations

Lemma

Average expectations are given by

\[
\begin{align*}
\bar{E}_t[U_t] &= (I - \Sigma)U_t \\
\bar{E}_t[V_{it}^1] &= -\Sigma_{v^1,u}U_t \\
\bar{E}_t[V_{it}^2] &= -\Sigma_{v^2,u}U_t
\end{align*}
\]

where

- \(\Sigma\) is the conditional variance of \(U_t\)
- \(\Sigma_{v^1,u}\) and \(\Sigma_{v^2,u}\) are the conditional covariances of \(V_{it}^1\) and \(V_{it}^2\) with \(U_t\)
Solution Algorithm: Equilibrium Response

\( \phi \), the equilibrium response to aggregate shocks, is then characterized by

\[
\phi' = \frac{(1 - r) \, \mathbf{1}' \, (I - \Sigma) \, [I - r \, (I - \Sigma)]^{-1}}{1 - \sigma_u \, [\sigma_b \, \gamma_b \Sigma v^1 u + \sigma_z \, \gamma_z \Sigma v^2 u] \, [I - r \, (I - \Sigma)]^{-1}}
\]

Through expectations of agg. shocks: the ‘right’ reasons

Through expectations of idio. shocks: the ‘wrong’ reasons
Results: $\rho_b = \rho_z = 0$

- Firms do not care about transitory idiosyncratic shocks.

\[ \phi' = (1 - r) \mathbf{1}' (I - \Sigma) [I - r (I - \Sigma)]^{-1} \]

- Pricing complementarities delay adjustment.
  - $r = 0 \Rightarrow p_t = \mathbf{1}' (I - \Sigma) U_t = \mathbf{1}' \bar{E}_t[U_t]$
  - $r > 0 \Rightarrow$ Larger real effects since $(1 - r) [I - r (I - \Sigma)]^{-1} << I$

- This case corresponds to existing results with persistent real effects e.g. Woodford(2001)
Results: $\rho_b$ and/or $\rho_z > 0$

- Firms adjust prices both for the right and the wrong reasons.

- Relative importance determined by the posterior covariance matrix $\Sigma$
  - $\Sigma$ small $\Rightarrow$ Precise posterior beliefs $\Rightarrow$ Responses mainly driven by expectations about aggregate shocks
  - $\Sigma$ close to $I$ $\Rightarrow$ Little info about aggregate shocks in signals $\Rightarrow$ Response driven by expectations about idiosyncratic shocks. Magnitude of response then depends on signal-to-noise ratio for the idiosyncratic component
Results: $\rho_b = \rho_z = \rho$, $\tilde{\sigma}_b = \tilde{\sigma}_z = 0$ and $\sigma_u \to 0$

- $\Sigma \left( \hat{\phi} \right) \to I$

- The impact of $u_{t-s}$ on $p_t$ is then given by

$$\phi_s = \frac{\rho(1-r)}{1 - r\rho}(1 - (r\rho)^s)$$

$$\lim_{s \to \infty} \phi_s = \frac{\rho(1-r)}{1 - r\rho}$$

- If $\rho = 1$, then prices ultimately respond fully to the aggregate shock

$$\phi_s = 1 - (r)^s$$

$$\lim_{s \to \infty} \phi_s = 1$$
Quantitative Results: Calibration

- Parameters borrowed from the literature $\psi = 2, \theta = 4$ and $\kappa = 1$
- $\sigma_u$ is set to .0036 (at a monthly frequency)
- Six remaining parameters: $\rho_b, \rho_z, \sigma_b, \sigma_v, \tilde{\sigma}_b, \tilde{\sigma}_z$
- Remaining 4 parameters are chosen to match
  - Relative price dispersion of $6 - 10\%$
  - Relative quantity dispersion of $25 - 30\%$
  - Price quantity correlation $\in (-0.2, 0)$
  - Autocorrelation of prices 0.65 at a monthly frequency
Quantitative Results: Effect of Persistence

- Higher persistence mutes effects of complementarity
Quantitative Results: Benchmark Model

- Transitory noise leads to rigidity on impact, but effect dies out very quickly
- Complete convergence takes a very long time
Quantitative Results: Period Length

Choice of period length has 2 effects

- Arrival rate of new information
  - Transitory noise \((\tilde{\sigma}_b, \tilde{\sigma}_z)\) at shorter frequencies can mitigate this
- Ability of firms to respond to new information
  - Time lag between arrival of information and when prices take effect
Extensions: Productivity Shocks

- Idiosyncratic productivity creates another source of dispersion
- With transitory demand/wage shocks, real effects are persistent
Extensions: Productivity Shocks

- Model I: Transitory demand shocks ⇒ Counterfactual price-qty correlation
- Model II: Larger demand shocks ⇒ Counterfactual quantity dispersion

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<td>0.25-0.30</td>
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Table: Relative Price and Quantity Moments with Productivity Shocks
Conclusion

- Micro-data on price dispersion and quantity moments
  - $\Rightarrow$ Persistent idiosyncratic demand shocks

- Incomplete information with market-generated signals
  - $\Rightarrow$ Firms attribute aggregate shocks to idiosyncratic factors
  - $\Rightarrow$ Respond to aggregate shocks for the ‘wrong’ reasons

- Implications for real effects
  - Persistence in idiosyncratic shocks $\Rightarrow$ Significant short-run adjustment and small but persistent real effects in the long run

- Tension between generating both price dispersion and large persistent real effects