Screening and Adverse Selection in Frictional Markets

Benjamin Lester  
Philadelphia Fed

Venky Venkateswaran  
NYU Stern

Ali Shourideh  
Wharton

Ariel Zetlin-Jones  
Carnegie Mellon University

March 11, 2016

Disclaimer: The views expressed here do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia.
Many important markets feature adverse selection (AS) and screening

- Examples: insurance, loans, some financial securities
Introduction

Many important markets feature *adverse selection (AS)* and *screening*

- Examples: insurance, loans, some financial securities

Most of these markets are characterized by *imperfect competition*
Introduction

Many important markets feature adverse selection (AS) and screening

- Examples: insurance, loans, some financial securities

Most of these markets are characterized by imperfect competition

A model of AS, screening contracts & imperfect competition is lacking

- Large empirical literature (and some theory)
- But typically restricts contracts and/or assumes perfect competition
Introduction

Many important markets feature adverse selection (AS) and screening

- Examples: insurance, loans, some financial securities

Most of these markets are characterized by imperfect competition

A model of AS, screening contracts & imperfect competition is lacking

- Large empirical literature (and some theory)
- But typically restricts contracts and/or assumes perfect competition

Essential to answer some important questions

- **Positive**: does market structure affect estimates of adverse selection?
Introduction

Many important markets feature adverse selection (AS) and screening

- Examples: insurance, loans, some financial securities

Most of these markets are characterized by imperfect competition

A model of AS, screening contracts & imperfect competition is lacking

- Large empirical literature (and some theory)
- But typically restricts contracts and/or assumes perfect competition

Essential to answer some important questions

- Positive: does market structure affect estimates of adverse selection?
- Normative: recent attempts to ↑ competition & transparency desirable?
This Paper

A tractable model of adverse selection, screening and imperfect competition
Key Ingredients

• Adverse Selection: sellers have private info about quality.

• A fraction $\mu h$ have quality $h$, the rest quality $\ell$.

• Screening: Buyers offer general menus of contracts.

• Price-quantity pairs: induce sellers to self-select.

• Imperfect Comp: sellers either receive 1 or 2 offers (Burdett-Judd).

• $P(2 \text{ offers}) = \pi$, $P(1 \text{ offer}) = 1 - \pi$.

• Contract offered before buyers know if seller has competing offer.

• Nests perfect competition ($\pi = 1$), monopsony ($\pi = 0$) as special cases.
Key Ingredients

- **Adverse Selection**: sellers have private info about quality.
  - A fraction $\mu_h$ have quality $h$, the rest quality $\ell$. 

Screening: Buyers offer general menus of contracts.

Price-quantity pairs: induce sellers to self-select.

Imperfect Comp: sellers either receive 1 or 2 offers (Burdett-Judd).

$P(2\text{ offers}) = \pi$, $P(1\text{ offer}) = 1 - \pi$.

Contract offered before buyers know if seller has competing offer.

Nests perfect competition ($\pi = 1$), monopsony ($\pi = 0$) as special cases.
Key Ingredients

- **Adverse Selection**: sellers have private info about quality.
  - A fraction $\mu_h$ have quality $h$, the rest quality $\ell$.

- **Screening**: Buyers offer general menus of contracts.
  - Price-quantity pairs: induce sellers to self-select.
Key Ingredients

- **Adverse Selection**: sellers have private info about quality.
  - A fraction $\mu_h$ have quality $h$, the rest quality $\ell$.

- **Screening**: Buyers offer general menus of contracts.
  - Price-quantity pairs: induce sellers to self-select.

- **Imperfect Comp**: sellers either receive 1 or 2 offers (Burdett-Judd).
  - $P(2 \text{ offers}) = \pi$, $P(1 \text{ offer}) = 1 - \pi$.
  - Contract offered before buyers know if seller has competing offer.
  - Nests perfect competition ($\pi = 1$), monopsony ($\pi = 0$) as special cases.
What we do

- New **techniques** → complete characterization of unique eqm
  - No issues with existence, off-path beliefs.

- Positive: predictions for distribution of contracts offered/traded
- Equilibrium can be pooling, separating, or a mixture of both
- Separation when adverse selection (AS) severe, competition high
- Pooling when AS mild, competition low
- Identifying AS requires knowledge of market structure

- Normative: welfare effects of more competition & better information
- AS severe: welfare $\cap$-shaped with ↑ competition. Otherwise: decreasing.
- Low comp: welfare $\cap$-shaped as ↑ transparency. Otherwise: decreasing.
- Competition interacts with IC constraints in non-monotonic fashion.
- ↑ competition/transparency desirable only when AS severe, competition low
What we do

- **New techniques** → complete characterization of unique eqm
  - No issues with existence, off-path beliefs.

- **Positive**: predictions for distribution of contracts offered/traded
  - Equilibrium can be pooling, separating, or a mixture of both
  - Separation when adverse selection (AS) severe, competition high
  - Pooling when AS mild, competition low
  - Identifying AS requires knowledge of market structure

- Normative: welfare effects of more competition & better information
  - AS severe: welfare $\cap$-shaped with $\uparrow$ competition. Otherwise: decreasing.
  - Low comp: welfare $\cap$-shaped as $\uparrow$ transparency. Otherwise: decreasing.
  - Competition interacts with IC constraints in non-monotonic fashion.
  - $\uparrow$ competition/transparency desirable only when AS severe, competition low
What we do

• New **techniques** $\rightarrow$ complete characterization of unique eqm
  - No issues with existence, off-path beliefs.

• **Positive:** predictions for distribution of contracts offered/traded
  - Equilibrium can be pooling, separating, or a mixture of both
  - Separation when adverse selection (AS) severe, competition high
  - Pooling when AS mild, competition low
  - Identifying AS requires knowledge of market structure

• **Normative:** welfare effects of more competition & better information
  - AS severe: welfare $\cap$-shaped with $\uparrow$ competition. Otherwise: decreasing.
  - Low comp: welfare $\cap$-shaped as $\uparrow$ transparency. Otherwise: decreasing.
  - Competition interacts with IC constraints in non-monotonic fashion.
  - $\uparrow$ competition/transparency desirable only when AS severe, competition low
Related Literature

Empirical

- Chiappori-Salanie ('00), Ivashina ('09), Einav et al. ('10,'12), ...

Adverse Selection and Screening

- Rothschild-Stiglitz ('76), Dasgupta-Maskin ('86), Rosenthal-Weiss ('84), Bisin-Gottardi ('06), GSW ('10)...
- Mirrlees ('71), Stiglitz ('77), ...

Imperfect Competition and Selection

- Burdett-Judd: Garrett, Gomes, and Maestri ('14)
- Hotelling: V-B & S-M ('99), Benabou-Tirole ('14), Townsend-Zhorin ('15), Weyl & co-authors...
ENVIRONMENT
Environment

Large number of buyers and sellers

- Each seller has 1 unit of divisible good
  - Good is of quality $i \in \{l, h\}$ with probability $\mu_i$
  - Seller: receives utility $c_i$ per unit of consumption.
  - Buyer: receives utility $v_i$ per unit of consumption.
Environment

Large number of buyers and sellers

- Each seller has 1 unit of divisible good
  - Good is of quality $i \in \{l, h\}$ with probability $\mu_i$
  - Seller: receives utility $c_i$ per unit of consumption.
  - Buyer: receives utility $v_i$ per unit of consumption.

- If seller gives up $x$ units in exchange for a transfer $t$, payoffs are
  - Seller: $t + (1 - x)c_i$
  - Buyer: $xv_i - t$
Environment

Large number of buyers and sellers

- Each seller has 1 unit of divisible good
  - Good is of quality $i \in \{l, h\}$ with probability $\mu_i$
  - Seller: receives utility $c_i$ per unit of consumption.
  - Buyer: receives utility $v_i$ per unit of consumption.

- If seller gives up $x$ units in exchange for a transfer $t$, payoffs are
  - Seller: $t + (1 - x)c_i$
  - Buyer: $xv_i - t$

- Assumptions
  - Gains from trade for both types: $v_h > c_h$ and $v_l > c_l$
  - ‘Lemons’ assumption: $v_l < c_h$
  - Adverse Selection: Only sellers know asset quality
Environment

Screening

- Buyers post arbitrary menus of exclusive contracts
- Screening menus intended to induce self-selection
Environment

Screening

- Buyers post arbitrary menus of exclusive contracts
- Screening menus intended to induce self-selection

Imperfect Competition through search frictions

- Each seller receives 1 offer w.p. $1 - \pi$ and 2 offers w.p. $\pi$
  - Can vary degree of competition with a single parameter, nesting extremes:
    - $\pi = 0$: monopsony.
    - $\pi = 1$: Bertrand/perfect competition.
Environment

Screening

- Buyers post arbitrary menus of exclusive contracts
- Screening menus intended to induce self-selection

Imperfect Competition through search frictions

- Each seller receives 1 offer w.p. $1 - \pi$ and 2 offers w.p $\pi$
  
  - Can vary degree of competition with a single parameter, nesting extremes:
    - $\pi = 0$: monopsony.
    - $\pi = 1$: Bertrand/perfect competition.

- Some terminology we’ll use:
  - Seller with 1 offer = “captive”
  - Seller with 2 offers = “non-captive”
Applications

Market for financial securities

- Buyers make offers to sellers (or issuers): price and quantity
- Sellers have private information about value

Loan markets

- Lenders make offers to borrowers: loan size and interest rate
- Borrowers have private information about default risk

Insurance markets

- Insurers make offers to potential customers: coverage and premium
- (Risk-averse) customers have private info about health/accident/death risk
Strategies

buyer: offers menu of contracts

• sufficient to consider two contracts \( z \equiv \{(x_l, t_l), (x_h, t_h)\} \)

\[ (IC_i) : \quad t_i + c_i(1 - x_i) \geq t_{-i} + c_i(1 - x_{-i}) \quad i \in \{l, h\} \]
Strategies

buyer: offers menu of contracts

- sufficient to consider two contracts \( \mathbf{z} \equiv \{(x_l, t_l), (x_h, t_h)\} \)

\[(IC_i) : \quad t_i + c_i(1 - x_i) \geq t_{-i} + c_i(1 - x_{-i}) \quad i \in \{l, h\} \]

seller: chooses a contract from available menus

- 1 offer (captive seller): chooses \((x_i, t_i)\) by incentive compatibility
- 2 offers (non-captive seller): chooses \((x_i, t_i)\) or \((x'_i, t'_i)\) by

\[
\chi_i(\mathbf{z}, \mathbf{z}') = \begin{cases} 0 & \text{if } t_i + c_i(1 - x_i) < t'_i + c_i(1 - x'_i) \\ \frac{1}{2} & \text{if } t_i + c_i(1 - x_i) = t'_i + c_i(1 - x'_i) \\ 1 & \text{if } t_i + c_i(1 - x_i) > t'_i + c_i(1 - x'_i) \end{cases}
\]
Equilibrium definition

A symmetric equilibrium is a distribution $\Phi(z)$ such that almost all $z$ satisfy,

1. *Incentive compatibility*:

$$t_i + c_i(1 - x_i) \geq t_{-i} + c_i(1 - x_{-i}) \quad i \in \{h, l\}$$

2. *Seller optimality*:

$$\chi_i(z, z')$$ maximizes her utility

3. *Buyers optimality*:

$$z \in \arg \max_z \sum_{i \in \{l, h\}} \mu_i \left[ 1 - \pi + \pi \int_{z'} \chi_i(z, z') \Phi(dz') \right] (v_i x_i - t_i) \quad (1)$$
Only Mixed Strategy Equilibrium for $\pi \in (0, 1)$

Why? Suppose a pure strategy equilibrium exists.

1. Buyers make strictly positive profits from some type
2. Buyers competing for this type with probability $\pi > 0$

Therefore,

$\Rightarrow$ Incentives to undercut

$\Rightarrow$ Equilibrium necessarily features dispersion
Characterization Strategy

Equilibrium described by non-degenerate distribution in 4 dimensions

Proceed in 4 steps
Characterization Strategy

Equilibrium described by non-degenerate distribution in 4 dimensions

Proceed in 4 steps

1. Show that menus can be summarized by a pair of utilities \((u_h, u_l)\)

   - Reduces dimensionality of problem to distributions in 2 dimensions
Characterization Strategy

Equilibrium described by non-degenerate distribution in 4 dimensions

Proceed in 4 steps

1. Show that menus can be summarized by a pair of utilities \((u_h, u_l)\)
   - Reduces dimensionality of problem to distributions in 2 dimensions

2. Show there is a strictly increasing map between \(u_l\) and \(u_h\)
   - Menus rank-ordered (Strict Rank Preserving)
   - Reduces problem to distribution in 1 dimension + a monotonic function
Characterization Strategy

Equilibrium described by non-degenerate distribution in 4 dimensions

Proceed in 4 steps

1. Show that menus can be summarized by a pair of utilities \((u_h, u_l)\)
   - Reduces dimensionality of problem to distributions in 2 dimensions

2. Show there is a strictly increasing map between \(u_l\) and \(u_h\)
   - Menus rank-ordered (Strict Rank Preserving)
   - Reduces problem to distribution in 1 dimension + a monotonic function

3. Construct SRP equilibrium
Characterization Strategy

Equilibrium described by non-degenerate distribution in 4 dimensions

Proceed in 4 steps

1. Show that menus can be summarized by a pair of utilities \((u_h, u_l)\)
   - Reduces dimensionality of problem to distributions in 2 dimensions

2. Show there is a strictly increasing map between \(u_l\) and \(u_h\)
   - Menus rank-ordered (Strict Rank Preserving)
   - Reduces problem to distribution in 1 dimension + a monotonic function

3. Construct SRP equilibrium

4. Show that constructed equilibrium is unique
A utility representation

Result

In all menus offered in equilibrium,

- **the low types trades everything**: \( x_l = 1 \)
- **IC\(_l\) binds**: \( t_l = t_h + c_l(1 - x_h) \)

- Reduces dimensionality of problem to distribution in 2 dimensions
A utility representation

Result

In all menus offered in equilibrium,

- the low types trades everything: $x_l = 1$
- $IC_l$ binds: $t_l = t_h + c_l(1 - x_h)$

- Reduces dimensionality of problem to distribution in 2 dimensions

Result

Equilibrium menus can be represented by $(u_h, u_l)$ with corresponding allocations

$$t_l = u_l \quad x_h = 1 - \frac{u_h - u_l}{c_h - c_l} \quad t_h = \frac{u_l c_h - u_h c_l}{c_h - c_l}$$
A utility representation

In all menus offered in equilibrium,

- the low types trades everything: \( x_l = 1 \)
- \( IC_l \) binds: \( t_l = t_h + c_l(1 - x_h) \)

- Reduces dimensionality of problem to distribution in 2 dimensions

Equilibrium menus can be represented by \((u_h, u_l)\) with corresponding allocations

\[
\begin{align*}
    t_l &= u_l \\
    x_h &= 1 - \frac{u_h - u_l}{c_h - c_l} \\
    t_h &= \frac{u_l c_h - u_h c_l}{c_h - c_l}
\end{align*}
\]

Since we must have \( 0 \leq x_h \leq 1 \),

\[
c_h - c_l \geq u_h - u_l \geq 0
\]
A utility representation

We define the marginal distributions:

\[ F_i(u_i) = \int_{z'} 1 \left[ t'_i + c_i (1 - x'_i) \leq u_i \right] d\Phi(z') \quad i \in \{h, l\} \]
A utility representation

We define the marginal distributions:

\[ F_i(u_i) = \int_{z'} 1 \left[ t_i' + c_i (1 - x_i') \leq u_i \right] d\Phi(z') \quad i \in \{h, l\} \]

Then, each buyer solves

\[ \Pi(u_h, u_l) = \max_{u_l \geq c_l, \ u_h \geq c_h} \sum_{i \in \{l, h\}} \mu_i \left[ 1 - \pi + \pi F_i(u_i) \right] \Pi_i(u_h, u_l) \]

s.t. \( c_h - c_l \geq u_h - u_l \geq 0 \)

with \( \Pi_l(u_h, u_l) \equiv v_l x_l - t_l = v_l - u_l \)

\( \Pi_h(u_h, u_l) \equiv v_h x_h - t_h = v_h - u_h \frac{v_h - c_l}{c_h - c_l} + u_l \frac{v_h - c_h}{c_h - c_l} \)
A utility representation

We define the marginal distributions:

\[ F_i(u_i) = \int_{z'} 1 \left[ t_i' + c_i (1 - x_i') \leq u_i \right] \, d\Phi(z') \quad i \in \{h, l\} \]

Then, each buyer solves

\[ \Pi(u_h, u_l) = \max_{u_l \geq c_l, \, u_h \geq c_h} \sum_{i \in \{l, h\}} \mu_i \left[ 1 - \pi + \pi F_i(u_i) \right] \Pi_i(u_h, u_l) \]

s. t. \[ c_h - c_l \geq u_h - u_l \geq 0 \]

with \[ \Pi_l(u_h, u_l) \equiv v_l x_l - t_l = v_l - u_l \]

\[ \Pi_h(u_h, u_l) \equiv v_h x_h - t_h = v_h - u_h \frac{v_h - c_l}{c_h - c_l} + u_l \frac{v_h - c_h}{c_h - c_l} \]

Need to characterize the two inter-linked distributions \( F_l \) and \( F_h \)!
Properties of Equilibrium

Result

\( F_l \) and \( F_h \) have connected support and are continuous.

- Except for a knife-edge case (see paper)
- Proof more involved than standard case because of interdependencies
### Properties of Equilibrium

#### Result

*F_l and F_h have connected support and are continuous.*

- Except for a knife-edge case (see paper)
- Proof more involved than standard case because of interdependencies

#### Result

*The profit function \( \Pi(u_h, u_l) \) is strictly supermodular.*

- Intuition: \( u_l \uparrow \Rightarrow \Pi_h \uparrow \Rightarrow \text{stronger incentives to attract high types} \)
- \( \Rightarrow U_h(u_l) \equiv \arg\max_{u_h} \Pi(u_h, u_l) \) is weakly increasing
## Properties of Equilibrium

### Result

\(F_l\) and \(F_h\) have connected support and are continuous.

- Except for a knife-edge case (see paper)
- Proof more involved than standard case because of interdependencies

### Result

The profit function \(\Pi(u_h, u_l)\) is strictly supermodular.

- Intuition: \(u_l \uparrow \Rightarrow \Pi_h \uparrow \Rightarrow\) stronger incentives to attract high types
- \(\Rightarrow \quad U_h(u_l) \equiv \arg\max_{u_h} \Pi(u_h, u_l)\) is weakly increasing

### Theorem (Strict Rank Preserving)

\(U_h(u_l)\) is a strictly increasing function.

- Weakly increasing because of super-modularity
- Strictly increasing, not a correspondence because \(F_l, F_h\) well-behaved
Strict Rank Preserving Equilibria

- Useful for characterization:
  - Ranking of equilibrium menus identical across types
  - Menus attract same fraction of both types $F_i(u_l) = F_h(U_h(u_l))$
  - Greatly simplifies our task: only have to find $F_i(u_l)$ and $U_h(u_l)$

- Immediate implications for outcomes:
  - Terms of trade positively correlated across types
  - Buyers don't specialize, trade with equal frequency across types
Strict Rank Preserving Equilibria

• Useful for characterization:
  • Ranking of equilibrium menus identical across types
  • Menus attract same fraction of both types $F_l(u_l) = F_h(U_h(u_l))$
  • Greatly simplifies our task: only have to find $F_l(u_l)$ and $U_h(u_l)$

• Immediate implications for outcomes:
  • Terms of trade positively correlated across types
  • Buyers don’t specialize, trade with equal frequency across types
Constructing Equilibria
What We Already Know

![Diagram showing the relationship between \( \pi \) and \( \mu_h \) with labels indicating Monopsony and Perfect Comp.]
Perfect comp and “severe adverse selection” ⇒ Pure strategy separating eq.
What We Already Know

Perfect comp and “mild adverse selection” ⇒ Mixed Strategy Eq.
What We Already Know

Monopsony and “severe adverse selection” $\Rightarrow$ No Trade with High Type
What We Already Know

Monopsony and “mild adverse selection” ⇒ Full Trade
What We Already Know

Cross Subsidizing or Not?

Cross Subsidizing: \( \Pi_l < 0 < \Pi_h \)

No Cross Subsidizing: \( \Pi_l, \Pi_h \geq 0 \)

(Monopsony)

(Perfect Comp)
Today:

- Construct equilibrium with $\mu_h < \bar{\mu}$ explicitly
- Briefly describe equilibrium with $\mu_h \geq \bar{\mu}$
Equilibrium Characterization

Today:

- Construct equilibrium with $\mu_h < \bar{\mu}$ explicitly
- Briefly describe equilibrium with $\mu_h \geq \bar{\mu}$

Terminology:

- “Separating eqm:” all contracts have $u_h > u_l$ (i.e., $x_h < x_l = 1$)
- “Pooling eqm:” all contracts have $u_h = u_l$ (i.e., $x_h \leq x_l = 1$)
- “Mixed eqm:” some separating offers, some pooling.
Conjecture & Confirm: Equilibrium with $\mu_h < \bar{\mu}_h$ is Separating

Remember the buyer’s problem:

$$\Pi(u_h, u_l) = \max_{u_l \geq c_l, \ u_h \geq c_h} \sum_{i \in \{l,h\}} \mu_i \left[1 - \pi + \pi F_i(u_i)\right] \Pi_i(u_h, u_l)$$

s. t. $c_h - c_l \geq u_h - u_l \geq 0$

with $\Pi_l(u_h, u_l) \equiv v_l x_l - t_l = v_l - u_l$

$\Pi_h(u_h, u_l) \equiv v_h x_h - t_h = v_h - u_h \frac{v_h - c_l}{c_h - c_l} + u_l \frac{v_h - c_h}{c_h - c_l}$
Conjecture & Confirm: Equilibrium with $\mu_h < \bar{\mu}_h$ is Separating

Marginal benefits vs costs of increasing $u_l$

$$\mu_l \pi f_l(u_l) \Pi_l + (1 - \pi + \pi F_l(u_l)) \left[ -\mu_l + \mu_h \frac{V_h - C_h}{C_h - C_l} \right] = 0$$

Boundary condition

$$F_l(c_l) = 0$$

$$F_l(\bar{u}_l) = 1 \rightarrow F_l(u_l)$$

Equal profit condition

$$[1 - \pi + \pi F_l(u_l)] \Pi(U_h, u_l) = \Pi = \mu_l (1 - \pi) (v_l - c_l) \rightarrow U_h(u_l)$$
Conjecture & Confirm: Equilibrium with $\mu_h < \bar{\mu}_h$ is Separating

Marginal benefits vs costs of increasing $u_l$

$$\mu_l \pi f_l(u_l) \Pi_l + (1 - \pi + \pi F_l(u_l)) \begin{bmatrix} -\mu_l + \mu_h \frac{V_h - C_h}{C_h - C_l} \\ \text{MB of more low types} \\ \text{MC} \\ \text{MB of relaxing IC} \end{bmatrix} = 0$$

Boundary condition

$$F_l(c_l) = 0 \quad F_l(\bar{u}_l) = 1 \quad \rightarrow \quad F_l(u_l)$$
Conjecture & Confirm: Equilibrium with $\mu_h < \bar{\mu}_h$ is Separating

Marginal benefits vs costs of increasing $u_l$

$$\mu_l \pi f_l(u_l) \Pi_l + (1 - \pi + \pi F_l(u_l)) \left[ -\mu_l + \mu_h \frac{V_h - C_h}{C_h - C_l} \right] = 0$$

Boundary condition

$$F_l(c_l) = 0 \quad F_l(\bar{u}_l) = 1 \quad \rightarrow \quad F_l(u_l)$$

Equal profit condition

$$[1 - \pi + \pi F_l(u_l)] \Pi(U_h, u_l) = \bar{\Pi} = \mu_l (1 - \pi) (v_l - c_l) \quad \rightarrow \quad U_h(u_l)$$
Conjecture & Confirm: Equilibrium with $\mu_h < \bar{\mu}_h$ is Separating

Marginal benefits vs costs of increasing $u_l$

$$\mu_l \pi f_i(u_l) \Pi_i + (1 - \pi + \pi F_i(u_l)) \begin{bmatrix} -\mu_l + \mu_h \frac{v_h - c_h}{c_h - c_l} \\ \text{MC} \\ \text{MB of relaxing IC}_l \end{bmatrix} = 0$$

Boundary condition

$$F_i(c_l) = 0 \quad F_i(\bar{u}_l) = 1 \quad \rightarrow \quad F_i(u_l)$$

Equal profit condition

$$[1 - \pi + \pi F_i(u_l)] \Pi(U_h, u_l) = \bar{\Pi} = \mu_l (1 - \pi) (v_l - c_l) \quad \rightarrow \quad U_h(u_l)$$

These conditions are necessary. In paper, show sufficiency.
Equilibrium

- Full Separation: $0 < x_h < 1$ a.e.
- Full Pooling: $x_h = 1$ a.e.
- Mix: pool below $\bar{u}$, separate above

More competition (higher $\pi$) $\rightarrow$ less pooling

- gains to cream-skimming increase in $\pi$
- milder adv sel (higher $\mu_h$) $\rightarrow$ more pooling

- increased incentives to trade with $h$

Price Dispersion Theorem

For every $(\pi, \mu_h)$ there is a unique equilibrium.
Cross-subsidization equilibrium may feature:

- **Full Separation**: $0 < x_h < 1$ a.e.
- **Full Pooling**: $x_h = 1$ a.e.
- **Mix**: pool below $\bar{u}$, separate above

More competition (higher $\pi$) $\rightarrow$ less pooling

- Gains to cream-skimming increase in $\pi$
- Milder adv sel (higher $\mu_h$) $\rightarrow$ more pooling

- Increased incentives to trade with $h$

**Price Dispersion Theorem**

For every $(\pi, \mu_h)$ there is a unique equilibrium.
Equilibrium

Cross-subsidization equilibrium may feature:
- Full Separation: \(0 < x_h < 1\) a.e.
- Full Pooling: \(x_h = 1\) a.e.
- Mix: pool below \(\bar{u}\), separate above

More competition (higher \(\pi\)) \(\rightarrow\) less pooling

- Gains to cream-skimming increase in \(\pi\)
- Milder adversarial selection (higher \(\mu_h\)) \(\rightarrow\) more pooling

Increased incentives to trade with \(h\)

Price Dispersion

Theorem

For every \((\pi, \mu_h)\) there is a unique equilibrium.
Equilibrium

Cross-subsidization equilibrium may feature:

- Full Separation: $0 < x_h < 1$ a.e.
- Full Pooling: $x_h = 1$ a.e.

Price Dispersion

Theorem

For every $(\pi, \mu_h)$ there is a unique equilibrium.
Cross-subsidization equilibrium may feature:

- **Full Separation**: $0 < x_h < 1$ a.e.
- **Full Pooling**: $x_h = 1$ a.e.
- **Mix**: pool below $\bar{u}_i$, separate above

Theorem:

For every $(\pi, \mu_h)$ there is a unique equilibrium.
Cross-subsidization equilibrium may feature:

- Full Separation: $0 < x_h < 1$ a.e.
- Full Pooling: $x_h = 1$ a.e.
- Mix: pool below $\bar{u}_i$, separate above

More competition (higher $\pi$) $\rightarrow$ less pooling

- gains to cream-skimming increase in $\pi$

Milder adv sel (higher $\mu_h$) $\rightarrow$ more pooling

- increased incentives to trade with $h$
Equilibrium

Cross-subsidization equilibrium may feature:

- Full Separation: \( 0 < x_h < 1 \) a.e.
- Full Pooling: \( x_h = 1 \) a.e.
- Mix: pool below \( \bar{u}_i \), separate above

More competition (higher \( \pi \)) \( \rightarrow \) less pooling

- gains to cream-skimming increase in \( \pi \)

Milder adv sel (higher \( \mu_h \)) \( \rightarrow \) more pooling

- increased incentives to trade with \( h \)

Theorem

For every \( (\pi, \mu_h) \) there is a unique equilibrium.
Comparison with Literature

- Huge literature on adverse selection with screening
  - Rothschild-Stiglitz, Riley, ..., Guerrieri-Shimer-Wright
- For the most part, competitive models with Bertrand-type structure
  \[ \Rightarrow \] infinite elasticity in response to deviations
- As a result, off-path beliefs become important
  - RS: All types who gain from a deviating menu show up
  - GSW: Only the type who gains the most shows up
- This paper: Mixed strategies \( \Rightarrow \) Finite elasticities
  \[ \Rightarrow \] No need to take a stand on off path beliefs.
Implications
Some Positive Implications

- Dispersion in prices and quantities, across and within types
Some Positive Implications

- Dispersion in prices and quantities, across and within types

- SRP $\Rightarrow$ buyers don’t target a specific type
  - terms of each contract correlated across offers
Some Positive Implications

- Dispersion in prices and quantities, across and within types

- SRP $\Rightarrow$ buyers don’t target a specific type
  - terms of each contract correlated across offers

- Structure of eqm depends on distribution of asset quality ($\mu_h$)
  - determines structure of eqm: separating, mixed, or pooling (Burdett-Judd)

Effect of adverse selection on outcomes depends on trading frictions ($\pi$) $\Rightarrow$ need to know trading frictions to identify info frictions.
Some Positive Implications

- Dispersion in prices and quantities, across and within types

- SRP $\Rightarrow$ buyers don’t target a specific type
  - terms of each contract correlated across offers

- Structure of eqm depends on distribution of asset quality ($\mu_h$)
  - determines structure of eqm: separating, mixed, or pooling (Burdett-Judd)

- Effect of adverse selection on outcomes depends on trading frictions ($\pi$)
  $\Rightarrow$ need to know trading frictions to identify info frictions.
Normative Implications

Ex-ante welfare

\[ W = \mu_I v_l + \mu_h [v_h X_h + c_h (1 - X_h)] \]

with \( X_h \equiv \int_{\tilde{u}_l}^{\bar{u}_l} x_h(u_l) \, d\hat{F}(u_l) \)

Are the following desirable?

- Increasing competition
- Increasing information/transparency

Focus on severe adverse selection (\( \mu_h < \bar{\mu}_h \)), show all these policies

Desirable or irrelevant at the extremes, i.e. \( \pi = 0 \) or \( \pi = 1 \)

But, can be undesirable in the interior, esp. for \( \pi_{\text{high}} \)
Normative Implications

Ex-ante welfare

\[ W = \mu_l v_l + \mu_h [v_h X_h + c_h (1 - X_h)] \]

with \( X_h \equiv \int_{\bar{u}_l}^{\tilde{u}_l} x_h(u_l) \ d\hat{F}(u_l) \)

Are the following desirable?

- Increasing competition
- Increasing information/transparency
Normative Implications

Ex-ante welfare

\[ W = \mu_l v_l + \mu_h [v_h X_h + c_h (1 - X_h)] \]
with \( X_h \equiv \int_{\tilde{u}_l}^{\bar{u}_l} x_h(u_l) \, d\hat{F}(u_l) \)

Are the following desirable?

- Increasing competition
- Increasing information/transparency

Focus on severe adverse selection (\( \mu_h < \bar{\mu}_h \)), show all these policies

- Desirable or irrelevant at the extremes, i.e. \( \pi = 0 \) or \( \pi = 1 \)
- But, can be undesirable in the interior, esp. for \( \pi \) high
Result

If $\mu_h < \bar{\mu}_h$, $W$ maximized at $\pi \in (0, 1)$. 
(Quick) Intuition

Requires understanding the interaction between competition and IC constraints.
Requires understanding the interaction between competition and IC constraints.

1. All else equal, increasing competition for low types causes:
   - buyers offer more utility to low types, which relaxes their IC constraint
   - allows for more trade with high types, which increases $W$.

Which effect dominates depends on amount of competition.

- First effect dominates when $\pi$ is small ($\Pi_h / \Pi_l$ small).
- Second effect dominates when $\pi$ is large ($\Pi_h / \Pi_l$ large).

Details

Changes policy lessons drawn from competitive models...
(Quick) Intuition

Requires understanding the interaction between competition and IC constraints.

1. All else equal, increasing competition for low types causes:
   ⇒ buyers offer more utility to low types, which relaxes their IC constraint
   ⇒ allows for more trade with high types, which increases $W$.

2. All else equal, increasing competition for high types causes:
   ⇒ buyers offer more utility to high types, which tightens IC constraint
   ⇒ less trade with high types, which decreases $W$.

Which effect dominates depends on amount of competition.

- First effect dominates when $\pi$ is small ($\Pi_h / \Pi_l$ small).
- Second effect dominates when $\pi$ is large ($\Pi_h / \Pi_l$ large).
Quick Intuition

Requires understanding the interaction between competition and IC constraints.

1. All else equal, increasing competition for low types causes:
   ⇒ buyers offer more utility to low types, which relaxes their IC constraint
   ⇒ allows for more trade with high types, which increases $W$.

2. All else equal, increasing competition for high types causes:
   ⇒ buyers offer more utility to high types, which tightens IC constraint
   ⇒ less trade with high types, which decreases $W$.

Which effect dominates depends on amount of competition.
(Quick) Intuition

Requires understanding the interaction between competition and IC constraints.

1. All else equal, increasing competition for low types causes:
   ⇒ buyers offer more utility to low types, which relaxes their IC constraint
   ⇒ allows for more trade with high types, which increases $W$.

2. All else equal, increasing competition for high types causes:
   ⇒ buyers offer more utility to high types, which tightens IC constraint
   ⇒ less trade with high types, which decreases $W$.

Which effect dominates depends on amount of competition.

- First effect dominates when $\pi$ is small ($\Pi_h/\Pi_l$ small).
- Second effect dominates when $\pi$ is large ($\Pi_h/\Pi_l$ large).
(Quick) Intuition

Requires understanding the interaction between competition and IC constraints.

1. All else equal, increasing competition for low types causes:
   - buyers offer more utility to low types, which relaxes their IC constraint
   - allows for more trade with high types, which increases $W$.

2. All else equal, increasing competition for high types causes:
   - buyers offer more utility to high types, which tightens IC constraint
   - less trade with high types, which decreases $W$.

Which effect dominates depends on amount of competition.

- First effect dominates when $\pi$ is small ($\Pi_h/\Pi_l$ small).
- Second effect dominates when $\pi$ is large ($\Pi_h/\Pi_l$ large).

Changes policy lessons drawn from competitive models...
Asset Purchases

Asset purchases proposed to help markets suffering from adverse selection

• Similar: government option (insurance markets), FAFSA (student loans)
Asset Purchases

Asset purchases proposed to help markets suffering from adverse selection

- Similar: government option (insurance markets), FAFSA (student loans)

Lessons from lit with competitive markets (Tirole, Guerrieri-Shimer):

1. Can only $\uparrow W$ if government overpays, loses money

2. But if government willing to do so, $\uparrow W$ for sure
Asset Purchases

Asset purchases proposed to help markets suffering from adverse selection

- Similar: government option (insurance markets), FAFSA (student loans)

Lessons from lit with competitive markets (Tirole, Guerrieri-Shimer):

1. Can only $\uparrow W$ if government overpays, loses money

2. But if government willing to do so, $\uparrow W$ for sure

Our model: neither result true when $\pi < 1$.

- Government losing money neither necessary nor sufficient for $\uparrow W$
Asset Purchases

Government: will purchase any quantity at $p \in [c_l, v_l]$.

Can be mapped into an *exogenous* lower bound for $u_l$

Government option never exercised.

1. Can be helpful and cost the government nothing.
2. Can be harmful even if it was free.
What About More/Better Information?

Examples

- Permitting insurance providers to discriminate based on observables
- Introducing credit scores in loan markets
- Requiring OTC market participants to disclose trades

Answer: desirability of more/better info depends on \((\pi, \mu_h)\)

Note: \(W\) is linear when \(\pi = 0\) and \(\pi = 1\) ⇒ no effect on welfare
What About More/Better Information?

Examples

• Permitting insurance providers to discriminate based on observables
• Introducing credit scores in loan markets
• Requiring OTC market participants to disclose trades

Allow principals to condition on more information
What About More/Better Information?

Examples

- Permitting insurance providers to discriminate based on observables
- Introducing credit scores in loan markets
- Requiring OTC market participants to disclose trades

Allow principals to condition on more information

- Can be mapped into a mean-preserving spread of $\mu_h$
- Need to compare $\mathbb{E}[W(\mu_h)]$ to $W(\mathbb{E}[\mu_h])$
- Desirability is about the sign of $W''(\mu_h)$
What About More/Better Information?

Examples

- Permitting insurance providers to discriminate based on observables
- Introducing credit scores in loan markets
- Requiring OTC market participants to disclose trades

Allow principals to condition on more information

- Can be mapped into a mean-preserving spread of $\mu_h$
- Need to compare $\mathbb{E}[W(\mu_h)]$ to $W(\mathbb{E}[\mu_h])$
- Desirability is about the sign of $W''(\mu_h)$

Answer: desirability of more/better info depends on $(\pi, \mu_h)$

- Note: $W$ is linear when $\pi = 0$ and $\pi = 1 \Rightarrow$ no effect on welfare
Desirability of information

- $\mu_h < \bar{\mu}_h : W$ convex (concave) for low (high) $\pi$
  $\Rightarrow$ more info desirable in concentrated markets, undesirable otherwise

- $\mu_h > \bar{\mu}_h : W$ is (weakly) concave for all $\pi$
  $\Rightarrow$ more info always undesirable
Conclusion

Lots of interest in markets where asymmetric info is a 1st order problem

- Insurance, loans, CDS, ...
Conclusion

Lots of interest in markets where asymmetric info is a 1st order problem

- Insurance, loans, CDS, ...

- Empirical: identifying adverse selection from terms/outcomes
Conclusion

Lots of interest in markets where asymmetric info is a 1st order problem

- Insurance, loans, CDS, ...
- Empirical: identifying adverse selection from terms/outcomes
- Theory: optimal intervention/regulation.
Conclusion

Lots of interest in markets where asymmetric info is a 1st order problem

- Insurance, loans, CDS, ...
- Empirical: identifying adverse selection from terms/outcomes
- Theory: optimal intervention/regulation.

Existing literature either restricts contracts or assumes perfect comp
Conclusion

Lots of interest in markets where asymmetric info is a 1st order problem

- Insurance, loans, CDS, ...
- Empirical: identifying adverse selection from terms/outcomes
- Theory: optimal intervention/regulation.

Existing literature either restricts contracts or assumes perfect comp

This paper:

1. Tractable model w/ AS, imperfect comp, sophisticated contracts
2. Many testable implications
   - info & trading frictions should be jointly identified
3. Novel normative implications
   - conclusions with $\pi = 1 \neq$ those with $\pi < 1$
Extra Stuff
Intuition

Theorem

$U_h(u_l)$ is a strictly increasing function.

Back to [Properties of Equilibrium]
**Theorem**

$U_h(u_l)$ is a **strictly increasing function**.
Intuition

Theorem

\( U_h(u_l) \) is a \textit{strictly increasing function}. 

\[ U_h(u_l) \]

\[ \hat{u}_h1 \]

\[ \hat{u}_h2 \]

\[ u_l \]

\[ \hat{u}_l \]
**Intuition**

**Theorem**

$U_h(u_l)$ is a *strictly increasing function*.

\[
Pr (u_h = \hat{u}_l) = \int_{\hat{u}_h1}^{\hat{u}_h2} f_h(u_h) du_h
\]
No cross-subsidization: Price vs quantity (conditional)

\[ \pi = 0.2 \]

\[ \pi = 0.5 \]

\[ \pi = 0.95 \]

Correlation \(<\ 0\) for suff. high \(\pi\)

A strategy to infer competitiveness?
Quotes

Einav, Finkelstein, and Levin

“There has been much less progress on [...] models of insurance contracting that incorporate realistic market frictions. One challenge is to develop an appropriate conceptual framework. Even in stylized models of insurance markets with asymmetric information, characterizing competitive equilibrium can be challenging, and the challenge is compounded if one wants to allow for realistic consumer heterogeneity and market imperfections.”

Or, as Chiappori et al (2006) put it:

“there is a crying need for...models...devoted to the interaction between imperfect competition and adverse selection”

Back to Introduction
Why is Welfare Hump-Shaped in $\pi$?

Because $x_h$ is hump-shaped in $\pi$ and $F_l$ is shifting right.
Why is Welfare Hump-Shaped in $\pi$?

Because $x_h$ is hump-shaped in $\pi$ and $F_l$ is shifting right.
Why is Welfare Hump-Shaped in $\pi$?

Because $x_h$ is hump-shaped in $\pi$ and $F_l$ is shifting right.

![Graph showing the relationship between $F_l(u_l)$ and $x_h(u_l)$ with the hump-shaped nature of $x_h$ and the shifting of $F_l$.](image)
Why is Welfare Hump-Shaped in $\pi$?

Because $x_h$ is hump-shaped in $\pi$ and $F_l$ is shifting right.
Why is $x_h(u_l)$ Hump-Shaped?

Two effects from competition:

1. buyers give more surplus to $l$ type sellers.
   - relaxes IC constraint $\rightarrow x_h \uparrow$

\[ U_h(u_l) \]
Why is $x_h(u_l)$ Hump-Shaped?

Two effects from competition:

1. buyers give more surplus to $l$ type sellers.
   - relaxes IC constraint $\rightarrow x_h \uparrow$

2. buyers give more surplus to $h$ type sellers.
   - tightens IC constraint $\rightarrow x_h \downarrow$

\[ U_h(u_l) \]
\[ \pi_h \]
\[ c_h \]
Why is $x_h(u_l)$ Hump-Shaped?

Two effects from competition:

1. buyers give more surplus to $l$ type sellers.
   - relaxes IC constraint $\rightarrow x_h \uparrow$

2. buyers give more surplus to $h$ type sellers.
   - tightens IC constraint $\rightarrow x_h \downarrow$

Which dominates? Depends on whether $U'_h(u_l) \lesssim 1$.

- i.e., whether buyers trying to attract more $l$ or $h$.
- this depends on relative profits $\frac{n_h}{n_l}$...
Severe Adverse Selection: Allocations

- Slope of $U_h$ determined by ratio of profits, $\Pi_h/\Pi_l$
Severe Adverse Selection: Allocations

- Slope of $U_h$ determined by ratio of profits, $\frac{\Pi_h}{\Pi_l}$
  - At low $u_l$, $\frac{\Pi_h}{\Pi_l}$ small, competition stronger for type-$l$, $U'_h(u_l) < 1$
  - At high $u_l$, $\frac{\Pi_h}{\Pi_l}$ large, competition stronger for type-$h$, $U'_h(u_l) > 1$